Optimal Taxation of Capital in the Presence of Declining Labor Share*

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We analyze the implications of the decline in labor's share in national income for optimal Ramsey taxation. It is optimal to accompany the decline in labor share by raising capital taxes only if the labor share is falling because of a decline in competition or other mechanisms that raise the share of pure profits. This result holds under various alternative institutional arrangements that are relevant for optimal taxation of capital income. A quantitative application to the U.S. economy shows that soaring profit shares since the 1980's can justify a significantly increasing path of capital income taxes.

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1 Introduction

This paper analyzes optimal capital and labor income taxation in an economy where the labor tax base is shrinking. This is quite relevant from the perspective of practical policymaking since there is strong support among economists for the view that the labor's share in national income (labor share hereafter) has been declining in many developed economies.[1] Figure 1 below depicts the labor share for the US economy for the post war era. The labor share is stable until the early 1980's and has been falling at a considerable rate since then. This trend implies that a government that, by following the suggestions of the classical Ramsey tax theory, sets capital income taxes to zero or at least keeps them low relative to labor income taxes, would experience a decline in overall tax revenue. Such a government would have to reform its tax system in order to make up for this decline. Interestingly, the academic literature has remained quite silent on the question of how we should reconsider the financing of government spending in response to declining labor share. In this paper, we take a step in this direction by using a standard macroeconomic model to investigate the optimal way of reforming the tax system along a transition exhibiting a decline in labor share under alternative assumptions regarding the availability of different fiscal policy tools.

We set up a representative agent neoclassical growth model in which there is a government that needs to finance an exogenous stream of expenditures using linear taxes on capital and labor income. We focus on a representative agent framework intentionally in order to avoid issues of inequality and redistribution. We allow for the possibility that firms may earn pure economic profits by modelling monopolistic competition in the product market. There is diversity of opinion in the literature on the mechanisms that are responsible for the decline in labor share. Because we do not want to take a stance on which mechanisms are more important before deriving qualitative results, our model incorporates virtually all of them.

By accounting, a decline in labor share is accompanied by a rise in capital share, a rise in profit share, or both. We categorize the theories of the decline in labor share proposed in the literature into two groups depending on whether they involve a rise in capital's or

[1]See Elsby, Hobijn, and Sahin (2013), Karabarbounis and Neiman (2014), Autor, Dorn, Katz, Patterson, and Reenen (2017), Barkai (2020), De Loecker, Eeckhout, and Unger (2020), Acemoglu and Restrepo (2019), and Farhi and Gourio (2018). There is controversy, however, about the degree to which the fall in the labor share is due to measurement issues such as the treatment of intangible capital (Koh, Santaeulalia-Llopis, and Zheng (2020)) or depreciation and production taxes (Bridgman (2018)) and the specificity of it to particular industries, mainly housing (Rognlie (2015)).

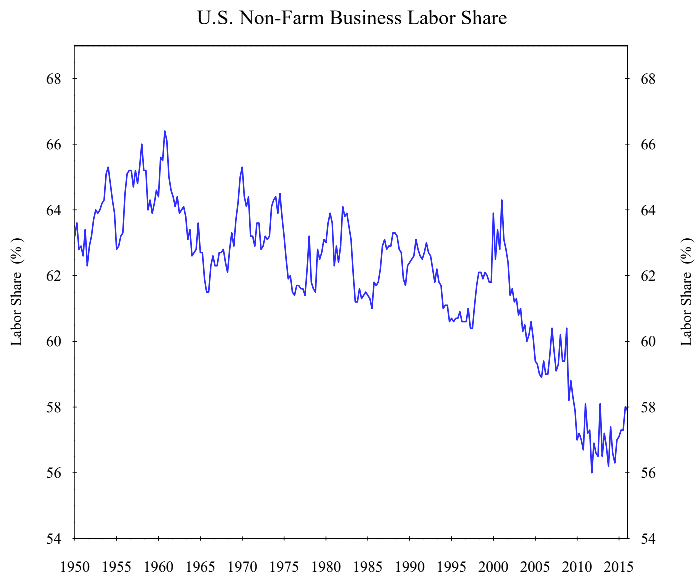


Figure 1: U.S. Non-Farm Business Labor Share

This figure depicts the evolution of U.S. non-farm labor share and is calculated using Bureau of Economic Analysis National Income and Product Accounts Tables.

profit's share of income. Rise in automation, capital augmenting technical change, decline in capital prices, offshoring of labor-intensive production and rising share of intangible capital are all theories of rise in capital share whereas declining competition is a theory of increasing profit share.[2] Our main qualitative finding is that the nature of the optimal tax reform for an economy that experiences a decline in labor share depends on whether this decline is accompanied by a rise in capital or profit share. If labor share is declining because production is becoming more capital intensive, then it is optimal to increase labor income taxes to make up for the loss in tax revenue. If, on the other hand, the decline in labor share coincides with a rise in profit share and profits cannot be not fully confiscated, then it is optimal to increase the tax rate on capital income.

Intuitively, whenever an economy is generating pure profits, it is optimal for the government to tax them away fully since taxing pure profits is non-distortionary. If taxing profit income at 100% is not an option for the government, an assumption maintained in the current paper, then it is optimal to tax factors that contribute to profit creation as this provides

[2]It is important to stress that we define profit income as firm earnings in excess of *all* production costs including fixed costs of production. For instance, positive net markups arising to break even in the presence of fixed capital costs would not generate pure economic profits. Therefore, we would categorize a theory that explains declining labor share using rising fixed capital costs of production as a rising capital share theory of declining labor share.

an *indirect* way of taxing profits. Obviously, capital is one such factor as it contributes to production, and hence, profit generation. There may be different reasons why a confiscatory profit tax may not be feasible in practice. One possibility is that although profit income can be distinguished from capital income, and hence can be taxed at a different rate, confiscation of profits is not feasible perhaps because firms can hide part of their profits from the tax authority. If this is the case, which we model in a reduced-form fashion by assuming that there is an exogenous upper bound on the profit tax rate, the aforementioned indirect tax motive is the sole motive for taxing capital in our model. Another possibility is that it might be hard for governments to distinguish profit income from capital income. We model this case by assuming a uniform tax rate that applies to both capital and profit income, and show that this introduces an additional motive to tax capital since a tax on capital income acts *directly* as a tax on profit income in the uniform tax case. Furthermore, when profits are generated due to the presence of market power as in our model, firms display inefficiently low investment (and labor) demand, which implies there is also a motive to subsidize investment (and labor). One possibility is to assume that these inefficiencies are dealt with at the firm level, where they originate, through product market interventions.[3] If the government does not have access to such product market policies, then, in addition to the direct and indirect tax motives explained above, a Pigouvian capital subsidy motive is also present.

The discussion above implies that depending on the institutional details regarding the set of fiscal policy tools that are available to the government, the optimal tax rate on capital is shaped by one, two or all of these three forces. For expositional purposes, we begin our theoretical characterization of the optimal tax rate with a baseline case in which all the three forces are at work. This is the case in which the government is restricted to choose a uniform tax rate that applies to both capital and profit income and there are no product market policies available to the government. In this case, in line with the discussion above, the optimal long-run capital tax formula consists of three components which reflect the indirect and the direct profit tax revenue benefits of capital taxation, and the Pigouvian subsidy. Then, we illustrate how institutional details affect optimal capital taxes by establishing their impact on the optimal long-run tax formula. Starting from the baseline case, if product

[3]This implementation is in line with a growing literature that studies how to design product market policies in correcting inefficiencies stemming from market power. See, among others, Edmond, Midrigan, and Xu (2018), Atkeson and Burstein (2019), and Boar and Midrigan (2020). Our paper differs from these papers in the sense that while our focus is on the optimal financing of government spending using income taxation they focus on correcting inefficiencies stemming from product market distortions.

market subsidies become available to the government, then the subsidy component disappears and the optimal tax rate is given by the direct and indirect tax components. If, in addition to the presence of product market subsidies, profit taxes can be set separately from capital taxes (but there is an exogenous upper bound on the profit tax rate), then the direct tax component also disappears and the optimal tax rate is given by the indirect tax component alone.[4] It is important to note that if profit taxes can be set separately and we do not impose an exogenous restriction on profit taxes, then the motives for taxing capital income described in this paper disappear irrespective of the sources of the decline in the labor share.

On the quantitative side, we first measure the changes in labor and capital shares for the US economy since the early 1980's. Our measurement implies a 15% increase in profit share during this period, which is consistent with notable contributions to the literature such as Barkai and Benzell (2018) and De Loecker, Eeckhout, and Unger (2020). We then calibrate the model to match the empirical evolution of income shares in the US economy assuming that profit share remains its current level of 15% in the long run. We first consider a hypothetical tax reform which the government carries out in the early 1980's foreseeing the upcoming trends in the underlying factors that give rise to the changes in income shares. In the baseline case, the optimal capital tax rate increases from zero to its long-run value of 8% by 2021 whereas the optimal labor tax rate is fairly smooth with a long-run value of 23%. In the implementation with product market subsidies, the optimal capital tax rate rises from zero to 36% by 2021, and converges to 38% in the long run. Both capital and labor taxes are higher relative to the baseline case because investment and labor demand are subsidized via product market policies instead. When profit taxes are set separately from capital taxes and product market subsidies are available, the optimal capital tax rate converges to its long-run level of about 7% by 2021. The optimal capital tax rate is lower than the uniform tax case since the direct tax channel is not present. In all cases, the optimal capital tax rate is low early on and increases with time because the profit share is around zero in the 1980's and rises with time. We also consider a tax reform that takes place in 2021. The optimal capital taxes already start around their long-run levels in all three cases since profit share is already 15% at the time the time of the reform. The optimal tax rates are higher relative to early 1980's reform because the government needs to finance a higher debt burden in 2021.

[4]There is also a fourth case that we do not investigate in this paper: one in which profit and capital taxes can be set separately but product market policies are not available to the government. Judd (2002) shows that the optimal long-run capital tax rate is negative for this case.

The optimal tax analysis discussed so far assumes that the government corrects monopolistic distortions, be it via subsidizing capital and labor income on the consumer side or via product market policies. However, in reality structural problems such as product market distortions might be hard to solve for various reasons. Motivated by this, we also analyze optimal taxation under the assumption that the government does not correct product market distortions, and discuss qualitative and quantitative properties of optimal taxes.

Related Literature. Our paper is related to several strands of literature, though, to the best of our knowledge, no other paper analyzes optimal tax design in the face of declining labor share. First, in our environment, the key to the optimality of increasing capital taxes is the rise in profit share. In this regard, an influential backdrop to our paper is Dasgupta and Stiglitz (1971) who show that, when there are pure profits due to decreasing returns to scale in production and profits cannot be taxed at 100%, it is optimal to tax intermediate inputs since taxing these provide an indirect tax on profits. Jones, Manuelli, and Rossi (1997) show that this logic implies the optimality of taxing capital in the long run in the neoclassical growth model. Judd (2002) shows that, when profits exist due to market power in the product market, the optimal long-run capital wedge is negative, calling for a subsidy. Guo and Lansing (1999) and Coto-Martinez, Garriga, and Sanchez-Losada (2007) question the generality of Judd (2002)'s subsidy result by considering restricted government policies and different economic environments.[5] Our paper differs from this literature as it analyzes an optimal tax reform in response to declining labor share. Our contribution lies in providing both qualitative lessons as to when capital taxes become a part of such a reform and a quantitative analysis of how strong the capital tax response should be. Our analysis also incorporates various institutional arrangements that are currently debated in policy circles, such as the use of product market policies, to the taxation of capital and labor income.

Second, there is a burgeoning literature that makes a case for taxation of robots. Following the skill premium literature, Slavik and Yazici (2014) assume a machine-skill complementarity. This implies that machines raise the marginal product of the skilled relative to the unskilled, and this increases inequality. It is, thus, desirable to deter the accumulation of machines from the perspective of a redistributive government. Using a quantitative model that features technical progress in automation and endogenous skill choice, Guerreiro, Re-

[5] There is also a new set of papers that analyze optimal redistributive labor income taxation in the presence of market power. See, for example, Eeckhout, Fu, Li, and Weng (2021).

belo, and Teles (2021) show that a similar argument implies the optimality of taxing robots when some currently active workers are locked into routine occupations.[6] In all these papers taxing robots is socially desirable because it is redistributive while we argue that taxing robots (or capital in general) provides a more *efficient* way of financing government's budget when capital accumulation contributes to creation of pure economic profits.[7] An exception is Acemoglu, Manera, and Restrepo (2020) who investigate optimal taxation in a task-based framework of automation assuming a representative agent. While we allow for the possibility of different mechanisms behind the decline of labor share, they focus on automation. Their quantitative analysis shows that the optimal capital tax rate is larger than the actual tax rate on capital, implying that the US tax system is biased in favor of capital.[8]

The organization of the article is as follows. Section 2 presents the model. Section 3 presents the theoretical characterizations of optimal taxes under various alternative institutional arrangements while Section 4 displays the corresponding quantitative results. Section 5 analyzes optimal taxation in the inefficient economy. Finally, Section 6 concludes.

2 Model

Consider a neoclassical growth model in which there is a representative consumer who lives forever. There are also firms that produce and sell intermediate and final goods. All firms are owned by the consumer. We introduce profits into our environment in the simplest possible manner: Dixit-Stiglitz monopolistic competition. This is the key departure of our model from the neoclassical growth model.[9] Finally, there is a benevolent government that needs

[6] See also Costinot and Werning (2018) and Thuemmel (2018) for a similar argument for robot taxation.

[7] There is also a growing literature on the optimal redistributive taxation of capital using quantitative models with rich heterogeneity and uninsured income risk. See, among others, Domeij and Heathcote (2004) and Conesa, Kitao, and Krueger (2009). The New Dynamic Public Finance literature, which has followed the seminal contribution of Golosov, Kocherlakota, and Tsyvinski (2003), also investigates optimal capital taxation in dynamic Mirrlesian private information models with idiosyncratic labor income shocks. Finally, see also Saez and Stantcheva (2018) who develop a theory of optimal redistributive capital taxation that expresses optimal tax formulas in sufficient statistics.

[8] It is optimal to tax capital in Acemoglu, Manera, and Restrepo (2020) because the authors assume that the government should balance its budget period by period. If one instead allows for a, perhaps more standard, intertemporal government budget, then one recovers the optimality of zero capital taxes in the long run in their environment. This is in line with our result that the capital-intensive theories of the decline in labor share such as automation do not per se justify taxing capital income.

[9] Although our exposition uses monopolistic competition to generate pure profits, the results of our model are more general. As we shall see in Section 3, independent of the microfoundation behind it, as long as labor share declines due to rising profit share, it is optimal to have rising capital income taxes.

to finance a given stream of public spending.

Final Good Producers. Firms that produce the final good are perfectly competitive and operate a constant elasticity of substitution (CES) production function that combines a unit measure of intermediate goods $y_{i,t}$. Taking prices of intermediate goods, $\xi_{i,t}$, as given, the problem of the representative final good producer is:

$$\max_{y_{i,t}} y_t - \int_0^1 \xi_{i,t} y_{i,t} di \qquad \text{s.t.} \qquad y_t = \left(\int_0^1 y_{i,t}^{\frac{\varepsilon_t - 1}{\varepsilon_t}} di \right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}}.$$

The first-order optimality condition of this problem with respect to $y_{i,t}$ gives the demand as a function of price for each intermediate good $y_{i,t} = y_t \xi_{i,t}^{-\varepsilon_t}$.

Intermediate Good Producers. Each intermediate good producer is a monopolistic competitor. Producer of intermediate good $y_{i,t}$ uses a CES technology, F_t, to combine capital and labor to produce the intermediate good. This firm solves:

$$\pi_{i,t} = \max_{\xi_{i,t}, y_{i,t}, k_{i,t}, l_{i,t}} \xi_{i,t} y_{i,t} - r_t k_{i,t} - w_t l_{i,t} \quad \text{s.t. } y_{i,t} = F_t(k_{i,t}, l_{i,t}) \text{ and } y_{i,t} = y_t \xi_{i,t}^{-\varepsilon_t}, \tag{1}$$

where r_t and w_t represent the real rental rate of capital and real wage rate, respectively. The problem of the intermediate good producer can be solved in two steps. In the first step, for a given marginal cost of producing the intermediate good, $m_{i,t}$, the firm chooses its price to maximize profits:

$$\max_{\xi_{i,t}} \xi_{i,t} y_{i,t} - m_{i,t} y_{i,t} \quad s.t. \quad y_{i,t} = y_t \xi_{i,t}^{-\varepsilon_t}. \tag{2}$$

The solution to this problem implies a constant markup over marginal cost: $\xi_{i,t} = m_{i,t} \frac{\varepsilon_t}{(\varepsilon_t - 1)}$. We focus on the symmetric equilibrium of the model in which all intermediate goods firms make identical choices of inputs and prices. This implies $y_{i,t} = y_t$ and $\xi_{i,t} = 1$ for all $i \in [0, 1]$. We, therefore, have the optimal marginal cost of producing one more intermediate good equals $m_{i,t} = m_t = 1 - \frac{1}{\varepsilon_t}$ for all firms.

In the second step, each firm chooses capital and labor to minimize the cost of production. The firms also make same input choices in the symmetric equilibrium, so we have $k_{i,t} = k_t$ and $l_{i,t} = l_t$. The marginal cost of producing one more unit of the intermediate good using capital or labor at the optimum equals $\frac{r_t}{F_{k,t}} = \frac{w_t}{F_{l,t}} = 1 - \frac{1}{\varepsilon_t}$, where $F_{k,t}$ is short-hand notation for $\frac{\partial F_t(k_l,l_t)}{\partial k_t}$ and $F_{l,t}$ is defined analogously. Therefore, the rental and wage rates are given by

$$r_t = \left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} \qquad \text{and} \qquad w_t = \left(1 - \frac{1}{\varepsilon_t}\right) F_{l,t}. \tag{3}$$

As long as ε_t is finite, the intermediate goods producers possess market power. This allows them to keep their sale prices above marginal cost by producing at below the socially efficient level, which gives rise to inefficiently low demand for investment and labor. This is why the rental rates of capital and labor are below the corresponding marginal products.

Income shares. Notice that since intermediate goods are used up in production, total income is given by the production of the final goods firm, y_t. Plugging $\xi_{i,t} = m_{i,t}\frac{\varepsilon_t}{(\varepsilon_t-1)}$ into (2) in the symmetric equilibrium implies that the total profit income generated by intermediate goods producing firms equals $\pi_t = \frac{1}{\varepsilon_t} y_t$. Thus, the share of profit income in total income in period t, denoted by $S_{\pi,t}$, equals $\frac{\pi_t}{y_t} = \frac{1}{\varepsilon_t}$. Using the rental rates given by (3) to compute the income shares of capital and labor renders $S_{k,t} \equiv \frac{r_t k_t}{y_t} = \left(1 - \frac{1}{\varepsilon_t}\right) \frac{F_{k,t} k_t}{y_t}$ and $S_{l,t} \equiv \frac{w_t l_t}{y_t} = \left(1 - \frac{1}{\varepsilon_t}\right) \frac{F_{l,t} l_t}{y_t}$. By construction, the profit share is identical across all firms in this economy, which is at odds with the empirical findings of De Loecker, Eeckhout, and Unger (2020) who document substantial heterogeneity across firm profit rates and its evolution over time. In Appendix D, we extend the monopolistic competition model into a multi-sector setting, which enables us to capture some broad facts on the distribution of profit rates, and analyze optimal tax implications of the decline in labor share in that setting.

Representative consumer. A unit measure of identical consumers, who live forever, are born in period one with $k_1 > 0$ units of physical capital and b_1 units of government debt. Taking prices as given, consumers choose their consumption, labor supply and saving allocations every period. Furthermore, they decide how to allocate their saving between physical capital and government bonds. Consumers have access to a linear technology that transforms one unit of the consumption good into $\frac{1}{q_t}$ units of the capital good in period t.

The period utility of an individual who consumes c units of consumption and supplies l units of labor equals $u(c, l)$. The utility function satisfies standard assumptions, $u_c, -u_{cc}, -u_l, -u_{ll} > 0$, where u_c and u_{cc} are short-hand notation for $\frac{\partial u(c,l)}{\partial c}$ and $\frac{\partial^2 u(c,l)}{\partial c^2}$, respectively, and u_l and u_{ll} are defined similarly. We also assume that utility is separable between consumption and labor, that is $u_{cl} = 0$. The separability assumption is not important for the main results of this paper, and is adopted merely to make the derivations of the optimal tax formulas simpler.[10] People discount future with a factor $\beta \in (0,1)$. Taking prices $\{p_t, r_t, w_t\}_{t=1}^{\infty}$, taxes $\{\tau_{k,t}, \tau_{l,t}, \tau_{\pi,t}\}_{t=1}^{\infty}$, and $k_1 > 0$ and b_1 as given, an individual chooses an allocation $(c,k,l) \equiv \{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ to solve the following problem:

$$\max_{c,k,l} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t) \qquad \text{s.t.}$$

$$\sum_{t=1}^{\infty} p_t \left(c_t + q_t k_{t+1}\right) \leq \sum_{t=1}^{\infty} p_t \left(w_t l_t (1-\tau_{l,t}) + \bar{r}_t k_t + \pi_t (1-\tau_{\pi,t})\right) + p_1 b_1, \tag{4}$$

where p_t is the period t price of the consumption good and $\bar{r}_t = q_t + (r_t - q_t\delta)(1-\tau_{k,t})$ is the after-tax gross rate of return to capital.[11] Looking at the right-hand side of the budget constraint above, we notice that the consumer has three sources of income: labor, capital and profit, each taxed at linear rates. Following the convention in actual tax systems, we allow for capital depreciation to be deducted from capital income tax base.

The first-order optimality conditions of the consumer's problem are:

$$\frac{\beta u_{c,t+1}}{u_{c,t}} = \frac{p_{t+1}}{p_t}, \tag{5}$$

$$p_t q_t = p_{t+1} \bar{r}_{t+1}, \tag{6}$$

$$\frac{u_{l,t}}{u_{c,t}} = -w_t (1-\tau_{l,t}). \tag{7}$$

Combining (5) and (6) with the rental rate of capital given by (3), we see that in equilibrium:

$$u_{c,t-1} q_{t-1} = \beta u_{c,t} \left[q_t + \left(\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t \right) (1-\tau_{k,t}) \right]. \tag{8}$$

Similarly, combining (7) with the wage rate given by (3), we have in equilibrium:

$$\left(1 - \frac{1}{\varepsilon_t}\right) F_{l,t} (1-\tau_{l,t}) u_{c,t} = -u_{l,t}. \tag{9}$$

Conditions (8) and (9) are going to be useful when defining optimal taxes in Section 3.

[10] Optimal long-run tax formulas for non-separable case are derived in Appendix A.7, which shows that while the capital tax formula remains the same as in the separable case the labor tax formula is modified.

[11] Notice that, as in Chari, Nicolini, and Teles (2020), the consumer's budget constraint (4) is given in Arrow-Debreu form. As long as a non-arbitrage condition regarding the after-tax returns to capital and government bonds holds - which must hold in equilibrium - sequential markets budget constraints in which the consumer explicitly chooses these two assets every period is equivalent to (4) which describes a one-time, beginning of time trade of consumption goods in future periods.

Government budget balance. Government uses capital, labor, and profit income taxes $\{\tau_{k,t}, \tau_{l,t}, \tau_{\pi,t}\}_{t=1}^{\infty}$ to finance an exogenous stream of spending $\{g_t\}_{t=1}^{\infty}$ and initial debt b_1.

$$\sum_{t=1}^{\infty} p_t g_t + p_1 b_1 \leq \sum_{t=1}^{\infty} p_t \left(w_t l_t \tau_{l,t} + (r_t - q_t \delta) k_t \tau_{k,t} + \pi_t \tau_{\pi,t} \right). \tag{10}$$

Resource feasibility. Aggregate resource feasibility requires that for all $t \geq 1$

$$c_t + q_t k_{t+1} + g_t = F_t(k_t, l_t) + (1 - \delta) q_t k_t. \tag{11}$$

Tax-Distorted Market Equilibrium. *Given (k_1, b_1) and $\{g_t\}_{t=1}^{\infty}$, a tax-distorted market equilibrium is a policy $\{\tau_{k,t}, \tau_{l,t}, \tau_{\pi,t}\}_{t=1}^{\infty}$, an allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$, a price system $\{p_t, r_t, w_t\}_{t=1}^{\infty}$ and profits $\{\pi_t\}_{t=1}^{\infty}$ such that: 1) Given policy and prices, allocation solves representative consumer's problem; 2) All firms maximize profits; 3) Markets for final and intermediate goods, capital, and labor clear; 4) Government's budget constraint is satisfied.*

3 Optimal Tax Analysis

Consider the problem of a government who needs to finance a given stream of public spending. We assume that there is an institution or a commitment technology through which the government can bind itself to a particular sequence of policies once and for all in period one. Once the policy is chosen, consumers and firms interact in capital, labor and goods markets according to the equilibrium defined earlier. The government is sophisticated enough that it predicts that different government policies lead to different behavior of economic agents, which then leads to different market equilibria. There are possibly many policy sequences that can finance a given stream of government spending. Among these, the benevolent government chooses the one that maximizes the representative consumer's welfare.

It is well-known that in optimal tax problems of this sort the government would like to set the tax rate on capital income in the very first period as high as possible since this tax is effectively a lump-sum tax on first period capital income. To make the problem interesting, we follow the literature and set the initial capital tax rate to an exogenous value, $\bar{\tau}_{k,1}$.

In this environment, it is optimal to tax profit income at 100% as taxing pure profits also acts like a lump-sum tax in the sense that it is non-distortionary. However, in reality, there may be a number of reasons for which a complete confiscation of profits may not be optimal or feasible. One possibility is that it is hard for governments to fully distinguish profit income from capital income. In such a world, even if the government is unrestricted in its choice of the profit tax rate, it would not find it optimal to set it to 100% as that could be detrimental for capital accumulation since profit tax rate applies to (part of) capital income as well. This is the assumption we will make in the baseline case by setting the tax rate on capital and profit income to be the same in every period, i.e., $\tau_{\pi,t} = \tau_{k,t}$ for all t. Another possibility is that the government can distinguish profit and capital income, and hence can set a separate tax rate for profit income, but for various reasons, is restricted to set this tax rate below a certain level. We investigate this case in Section 3.2.

Under the assumptions outlined so far, given (k_1, b_1) and $\{g_t\}_{t=1}^{\infty}$, the government chooses $\tau = \{\tau_{k,t+1}, \tau_{l,t}\}_{t=1}^{\infty}$ to maximize the welfare of the representative consumer subject to the fact that the tax system $\{\tau_{k,t}, \tau_{l,t}, \tau_{\pi,t}\}_{t=1}^{\infty}$, the allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ and the price system $\{p_t, r_t, w_t\}_{t=1}^{\infty}$ constitute a market equilibrium.[12] Following the literature, in what follows, we often refer to the optimal tax problem as the Ramsey problem.

We follow the primal approach and first show that the Ramsey problem is equivalent to a planning problem where the government chooses an allocation subject to a number of conditions that summarize all the restrictions that are implied on allocations by the tax-distorted market equilibrium. This is achieved by Proposition 1 which establishes that the resource feasibility constraint together with two other constraints characterizes the tax-distorted market equilibrium completely.

Proposition 1. *If an allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ is part of a tax-distorted market equilibrium, then it satisfies the resource feasibility constraint (11), and the constraints (12) and (13) below. Conversely, suppose an allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ satisfies (11), (12) and (13). Then, we can construct prices and taxes such that this allocation together with constructed prices and taxes constitute an equilibrium allocation.*

[12]If the competitive equilibrium associated with each policy is unique, the objective of the Ramsey problem is well-defined. Unfortunately, it is well known that establishing uniqueness - and even existence - of competitive equilibria for economies with distortions can be quite hard in general. This difficulty applies to the case of the neoclassical growth model with tax distortions as well (see, for instance, Greenwood and Huffman (1992) and Jones and Manuelli (1999)). Given this, if there are multiple competitive equilibria associated with some policies, our definition of the optimal tax problem requires that a selection be made from the set of competitive equilibria. We follow the approach suggested by Chari and Kehoe (1999) who impose that whenever there is multiple equilibria corresponding to a policy, we select the equilibrium that yields the highest utility for the representative agent.

$$\sum_{t=1}^{\infty} \beta^{t-1} \left(u_{c,t} c_t + u_{l,t} l_t\right) = \sum_{t=1}^{\infty} \beta^{t-1} u_{c,t} \pi_t (1 - \tau_{\pi,t}) + u_{c,1}(\bar{r}_1 k_1 + b_1), \tag{12}$$

$$\tau_{\pi,t} = 1 - \frac{\frac{u_{c,t-1} q_{t-1}}{\beta u_{c,t}} - q_t}{\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t}, \quad \forall t \geq 2. \tag{13}$$

Proof. Relegated to Appendix A.1. □

The constraint (12) is called the implementability constraint, and is a standard constraint and a version of this is present in all Ramsey tax problems. The constraint (13) represents the restriction that profit and capital income tax rates are equal, and follows from (8).

Next, we characterize the allocation that solves the planning problem, namely the Ramsey allocation, by a set of optimality conditions. After that, we will back out optimal tax rates by comparing the optimality conditions of the Ramsey problem with those from the tax-distorted market equilibrium.

Ramsey problem. Given (k_1, b_1), initial policies $\tau_{\pi,1} = \tau_{k,1} = \bar{\tau}_{k,1}$, and a sequence of government spending, $\{g_t\}_{t=1}^{\infty}$, the government chooses allocation (c, k, l) to solve the problem:

$$\max_{c,k,l} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t) \qquad \text{s.t.} \tag{14}$$

$$(11), (12), \text{ and } \tau_{\pi,t} \text{ is given by } (13).$$

The first term on the right-hand side of the implementability constraint, which is equal to the net present value (NPV) of after-tax profit income, is the main difference to standard Ramsey problems without profits. The implementability constraint binds in the direction that the left-hand side should be greater than the right-hand side. In fact, an explicit derivation of this constraint from the government's budget constraint would reveal that the left-hand side corresponds to government's tax revenues while the right-hand side corresponds to its spending. As such, the NPV of after-tax profits appear as a cost in the Ramsey problem. Intuitively, since taxing pure profits is not distortionary, the Ramsey planner would like to confiscate them fully. When this is not possible, this is identical to a case where the government taxes profits at 100% but needs to rebate $(1 - \tau_{\pi,t})$ back to consumers.

Letting $\beta^{t-1}\mu_t$ and λ be the Lagrangian multipliers on period t feasibility constraint and the implementability constraint, respectively, and the star allocation denote the Ramsey allocation, the first-order optimality condition for capital in any period $t \geq 2$ is:

$$(k_t) : -\mu^*_{t-1} q_{t-1} + \beta \mu^*_t \left(F^*_{k,t} + (1-\delta) q_t \right) - \lambda^* \beta u^*_{c,t} \left[(1-\tau^*_{\pi,t}) \frac{\partial \pi^*_t}{\partial k_t} + \frac{\partial (1-\tau^*_{\pi,t})}{\partial k_t} \pi^*_t \right] = 0. \quad (15)$$

Pigouvian subsidy. The first two terms in (15) are standard, and represent the period $t-1$ physical cost of investing in period t capital stock and the period t physical return to that investment, respectively. A comparison of these terms to the equilibrium capital accumulation condition (8) reveals that the social physical return to capital, $F^*_{k,t}$, exceeds the private physical return to capital in the laissez-faire equilibrium, $(1 - 1/\varepsilon_t) F^*_{k,t}$. This distortion arises due to the presence of monopolistic competition in the product market and implies the optimality of a Pigouvian subsidy on capital accumulation.

The existence of untaxed profits in the implementability constraint, (12), introduces two new terms into the first-order condition of capital which are the terms in the square bracket in (15). We now discuss these terms.

Indirect tax on profit income. The first term is $-\lambda^* \beta u^*_{c,t} (1-\tau^*_{\pi,t}) \frac{\partial \pi^*_t}{\partial k_t}$. Increasing capital in period t increases the NPV of after-tax profits. This tightens the implementability constraint, and as such, represents an additional cost of increasing capital. The rise in the NPV of after-tax profits equals the rise in after-tax profits in period t, $(1-\tau^*_{\pi,t}) \frac{\partial \pi^*_t}{\partial k_t}$, times the shadow price of consumption, $\beta u^*_{c,t}$. The multiplier on the implementability constraint, λ^*, measures the social value of an additional unit of public funds. This social cost term implies a tax on capital income. Intuitively, since taxing profits is a lump-sum tax, government would like to tax profits away completely. When this is not possible, it is optimal to tax intermediate goods, capital in this case, since it acts as an *indirect* tax on profit income.

Direct tax on profit income. The second term is $-\lambda^* \beta u^*_{c,t} \frac{\partial (1-\tau^*_{\pi,t})}{\partial k_t} \pi^*_t$. It follows from (13) that a higher level of capital stock is consistent with a lower profit tax rate in equilibrium since $\frac{\partial (1-\tau^*_{\pi,t})}{\partial k_t} > 0$. This implies that increasing capital has an additional cost of increasing the NPV of after-tax profits by increasing the retention rate on profits in period t. The term $\lambda^* \beta u^*_{c,t}$ again translates this cost into social value of public funds. This additional social cost of increasing capital introduces another reason for its taxation. Intuitively, since capital and profit income are taxed at the same rate, taxing capital income provides a *direct* way of taxing profit income, and this is beneficial since taxing profits is non-distortionary.

Optimal taxes in the baseline case. Optimal capital and labor tax rates that implement the Ramsey allocation in the baseline market equilibrium defined in Section 2 can be computed by substituting the Ramsey allocation in equilibrium conditions (8) and (9), respectively. Furthermore, by comparing the first-order optimality conditions of the Ramsey problem (14) with (8) and (9), we can provide optimal tax formulas. Below, we provide such an optimal capital tax formula focusing on the steady state as the transition formula is complicated and does not provide much further understanding of the forces behind optimal taxes.[13] In the rest of the paper, we assume that the Ramsey allocation and the multiplier associated with the resource constraint (11) converges to a steady state. Straub and Werning (2020) showed that this assumption is not innocuous by proving that if the Ramsey planner faces an exogenous upper bound on the capital tax rate, then one can find a set of conditions under which the multiplier does not converge. We differ from them in that we do not assume an exogenous upper bound on the tax rate on capital income. However, even in this case, it is hard to prove the convergence of the Ramsey allocation under general preference specifications. Chari, Nicolini, and Teles (2020) establish the convergence of the Ramsey allocation for a preference specification that is widely used in the Macroeconomics literature and that we use in our quantitative part. In what follows, variables with no time subscript denote steady-state variables.

Proposition 2. *Suppose $\tau_{k,t} = \tau_{\pi,t}$ for all $t \geq 1$. The long-run optimal tax rate on capital income is given by*

$$\frac{\tau_k^*}{1-\tau_k^*} = \frac{S_\pi F_k^*}{F_k^* - \delta q}\left(-1 + \chi^*\left[1 + \mathcal{E}^*_{1-\tau_k,k}/\tilde{S}_k^*\right]\right) \tag{16}$$

where $\chi^* = \frac{\lambda^* u_c^*}{\mu^*}$ is the relative social value of public funds, $\mathcal{E}^*_{1-\tau_k,k} = \frac{\partial ln(1-\tau_k)}{\partial k}|_{k=k^*}$ is the elasticity of the retention rate on capital income with respect to equilibrium capital stock, and $\tilde{S}_k^* = \frac{F_k^* k^*}{Y^*}$ measures capital intensity of production, all evaluated at the steady state.

Proof. Relegated to Appendix A.2. □

[13] The presence of after-tax profit income alters optimal taxes on labor income as well. We provide the full characterization of the Ramsey allocation, which includes the first-order optimality conditions with respect to consumption and labor in Appendix A.2, along with the long-run optimal labor tax formula.

Interpreting the capital tax formula. The optimal capital tax rate given by (16) is the summation of three terms given in the paranthesis, all multiplied by the same factor, $\frac{S_\pi F_k^*}{F_k^* - \delta q}$. The first term, -1, corresponds to the Pigouvian subsidy on investment while the second and the third terms, χ^* and $\chi^* \mathcal{E}^*_{1-\tau_k,k}/\tilde{S}^*_k$, correspond to the indirect and direct profit tax revenue benefits of taxing capital. Observe that the larger is the sensitivity of the retention rate on capital with respect to equilibrium capital stock the higher is the capital tax rate. This is intuitive since when $\mathcal{E}^*_{1-\tau_k,k}$ is large, equilibrium capital stock is less sensitive to changes in the capital tax rate, which means we can increase profit and hence capital tax rate without distorting capital accumulation much. Similarly, it is decreasing in $\tilde{S}^*_k$: the more capital intensive production is the greater is the deadweight loss created by capital taxation. These revenue benefits of capital taxation accrue in terms of higher government revenues. As such, they must be weighted by the social value of public funds, $\lambda^* u^*_c$. On the other hand, the cost of taxing capital, which is the deadweight loss associated with slowing down capital accumulation, accrues in terms of lower output. The social cost of a unit decline in output equals the multiplier on the resource constraint, μ^*. The term, χ^*, which we call the relative social value of public funds, translates the revenue benefit into the same unit as the deadweight loss, that is foregone output. The common factor $\frac{S_\pi F_k^*}{F_k^* - \delta q}$ reveals the importance of the profit share for the magnitude of capital taxes. The part $\frac{F_k^*}{F_k^* - \delta q}$ would vanish if all of capital income was taxed and not just the part net of depreciation.[14] Finally, it is important to keep in mind that most of the terms in the formula are endogenous, and as such, are themselves affected by the tax policy.

Benchmark: zero profit income. A glance in the optimal tax formula given by (16) reveals that whenever there are no profits, we recover the classical Chamley-Judd result that the optimal capital tax is zero in the long run. Corollary 1 summarizes this result.

Corollary 1. *If $S_\pi = 0$ (equivalently $\varepsilon = \infty$), then $\tau_k^* = 0$.*

Proposition 3 below, which echoes another classical finding in the Ramsey literature (see, Chari and Kehoe (1999), among others), establishes a more general result about the optimality of not taxing capital that holds along the transition as well.

[14] If one also assumes that the production function satisfies the knife-edge case of Cobb-Douglas, $F(k,l) = k^\alpha l^{1-\alpha}$, then $\mathcal{E}^*_{1-\tau_k,k} = 1-\alpha$ and $\tilde{S}^*_k = \alpha$. In this case, the optimal capital tax formula reduces to $\frac{\tau_k^*}{1-\tau_k^*} = S_\pi \left(-1 + \chi^* \alpha^{-1}\right)$, which shows that, for a given level of government financing needs, χ^*, it is the profit share and the capital intensity that determines the optimal tax rate.

Proposition 3. *Suppose $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - v(l)$, where $v', v'' > 0$. If in some period $t \geq 3$, we have $S_{\pi,t-1} = S_{\pi,t} = S_{\pi,t+1} = 0$ (or equivalently $\varepsilon_{t-1} = \varepsilon_t = \varepsilon_{t+1} = \infty$), then, $\tau^*_{k,t} = 0$.*

Proof. Relegated to Appendix A.3. □

Assuming the same preference structure, Straub and Werning (2020) show that it is optimal to set the tax rate on capital income to the upper bound for a period of time (which can be indefinite). In the absence of exogenous upper bounds, our Ramsey planner finds it optimal to set the capital income tax rate to a very large number in the first period in which it is a choice variable ($t = 2$) and then set it to zero from next period on.[15]

Discussion. An important implication of Proposition 3 is that if we live in a competitive economy and the decline in labor share occurs due to a rise in capital share, then lessons from the classical Ramsey tax theory apply: it is optimal to set the capital tax rate to zero after a high initial tax and finance government spending with (higher) taxes on labor income. It may be useful to discuss which of the various theories of the decline in labor share call for optimal taxation of capital in our framework in light of this benchmark result. Elsby, Hobijn, and Sahin (2013) provide an account of US production becoming more capital-intensive due to offshoring of the production of labor-intensive goods while Karabarbounis and Neiman (2014) present a capital deepening story due to declining price of investment. According to the theory presented here, these do not call for taxation of capital. Philippon (2019), Barkai (2020), and De Loecker, Eeckhout, and Unger (2020) argue that the decline in competition and the resulting surge in profits is a major contributer to the decline in labor share, which would justify optimal taxes on capital. Rognlie (2015) attributes most of the decline in labor share in the US to rising non-labor income share in the housing sector. The model he uses to make sense of this empirical finding is one in which rising income in the housing sector accrues as competitive factor income to capital. In this case, again, our motives to tax capital do not apply.[16] Finally, Koh, Santaeulalia-Llopis, and Zheng (2020) attribute the decline

[15] In standard Ramsey problems, the planner sets a high tax rate on the capital income received in period two because this raises consumption in the first period, which reduces the shadow price of period one consumption, $u^*_{c,1}$, which reduces the right-hand side of (12). For the same reason, it is optimal to subsidize labor income in the first period. When there is a binding upper bound on the capital tax rate, this logic implies that the capital tax rate be set to the bound for a number (possibly infinite) of periods.

[16] Cette, Koehl, and Philippon (2019) show that after accounting for housing sector and self employment, the decline in labor share vanishes for many European countries but remains true for the US, since 2000. They claim that this difference between the US and Europe may single out the decline in competition and the resulting rise in corporate profits in the US as an explanation for declining US labor share.

in labor share to a change in the treatment of intangible capital in NIPA accounts and the rise in intangible capital share. To the extent that the rising intangible income accrues as competitive factor income, our motive to tax capital would not apply. If, on the other hand, the rise in income in the housing sector or the rise in intangible capital's share accrue partly as economic rents, then our motives to tax capital would apply.

3.1 Optimal Taxes with Product Market Interventions

In this section, we lay out an alternative implementation of the Ramsey allocation defined by the solution to (14) in which the problem of insufficient demand for capital and labor is dealt with in the product market via sales subsidies (instead of subsidizing consumer savings). We believe that this institutional design with product market policies is highly relevant since (i) it is precisely the product market where insufficient demand for capital and labor originates and (ii) product market subsidies are relatively easy to implement and are used in practice.[17] The use of product market policies is also the topic of a growing literature that studies how to design such policies to correct inefficiencies stemming from market power.[18]

The level of period t sales subsidy is set to $\tau_{s,t} = \frac{1}{\varepsilon_t - 1}$ in order to exactly offset the lack of demand for capital and labor that stems from monopolistic distortions. This guarantees that the rental rates equal their marginal products in equilibrium. Appendix A.4 provides a full description of the product market policy and formally proves the claim that it allows for a decentralization of the Ramsey allocation in a market environment where the interest and the wage rates equal the marginal products of capital and labor. Combining the version of equilibrium condition (8) in which the rental rate of capital $F^*_{k,t}(1 - 1/\varepsilon_t)$ is replaced by $F^*_{k,t}$ with (15), and evaluating at the steady state gives the following capital tax formula.

Proposition 4. *Suppose $\tau_{k,t} = \tau_{\pi,t}$ and $\tau_{s,t} = \frac{1}{\varepsilon_t - 1}$ for all $t \geq 1$. The long-run optimal tax rate on capital income is given by*

$$\frac{\tau^*_k}{1 - \tau^*_k} = \frac{S_\pi F^*_k}{F^*_k - \delta q} \chi^* \left[1 + \mathcal{E}^*_{1-\tau_k, k} / \tilde{S}^*_k \right]. \tag{17}$$

[17] Correcting monopolistic distortions at the firm level may also be desirable since in reality there may be heterogeneity in market power among firms, and correcting firm-specific wedges may not be possible by subsidizing consumers' savings.

[18] See, among others, Atkeson and Burstein (2019), Edmond, Midrigan, and Xu (2018), Boar and Midrigan (2020). The focus of the current paper is quite different from these papers since while they investigate correcting inefficiencies stemming from product market distortions we analyze optimal financing of government spending through income taxation.

Proof. Relegated to Appendix A.4. □

The main difference of the optimal capital tax rate with product market interventions given by (17) relative to the optimal capital tax rate without such policies given by (16) is that in (16) there is an additional term in the paranthesis, -1, that calls for a subsidy on capital income. This component is absent in (17) since in this case monopolistic distortions are dealt with at the firm level where they originate via product market subsidies. For this reason, the optimal capital tax rate is higher in the case with product market policies.

3.2 Optimal Taxation with Exogenous Profit Taxes

The optimal tax analysis so far assumed that the tax rate on capital and profit income has to be the same. In this section, we derive optimal tax formulas for the case where profit income is allowed to be taxed at a different rate than capital income. If we allow the government to choose the tax rate on profit income freely, it would optimally confiscate profits fully since profit tax is a lump-sum tax in our environment. In reality, however, taxing profits away at 100% may not be desirable or feasible for reasons that are not captured by our model such as the possibility that firms may be able to hide part of their profits. For this reason, we are going to set an exogenous upper limit on the profit tax rate. We maintain the assumption that the government uses product market policies to correct monopolistic distortions.

The Ramsey problem for the case with exogenous profit taxes is identical to (14) except that now the profit tax is exogenously given. As a result, the problem does not have constraint (13) any more. The explicit statement of the Ramsey problem and the associated first-order optimality conditions are deferred to Appendix A.5. Assuming that the economy, including the exogenous upper limit on profit tax rate, $\{\bar{\tau}_{\pi,t}\}_{t=1}^{\infty}$ converges to a steady state, we have the following long-run capital tax result.

Proposition 5. *Suppose there is a sequence of exogenous upper limits, $\{\bar{\tau}_{\pi,t}\}_{t=1}^{\infty}$, on profit taxes and suppose these limits converge to $\bar{\tau}_{\pi}$ over time. The long-run optimal tax rate on capital income is given by*

$$\tau_k^* = \frac{S_\pi F_k^*}{F_k^* - \delta q}\chi^*(1 - \bar{\tau}_\pi). \tag{18}$$

Proof. Relegated to Appendix A.5. □

The main difference between the optimal capital tax formula given by (18) and the optimal tax formula in the case with uniform tax rate on capital and profit income given by

(17) is that the term that represents the direct profit tax revenue benefit of taxing capital is absent in (18). This is intuitive: raising capital income tax rate does not directly raise profit tax revenues since the two tax rates are separate. Taxing capital acts as a tax on profit income only through the indirect tax channel, the magnitude of which is now controlled by the exogenous tax rate on profit income, $\bar{\tau}_\pi$.

4 Quantitative Analysis

This section presents the calibration of the model and the results of optimal tax simulations.

4.1 Calibration: Initial Steady State

We choose the parameters of the model so that the initial steady state of the model economy matches the early 1980's U.S. economy along selected key moments. The model is calibrated annually and the full set of parameters, targets and sources are summarized in Table 1.

Preferences. The discount factor β is set to 0.96 so that the model implied interest rate is equal to 4.1% (Atkeson and Kehoe (2005)). This implies a capital-output ratio of 2.5. The momentary utility function of the household takes the form

$$u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{l^{1+\phi}}{1+\phi}.$$

The constant elasticity of intertemporal substitution (CEIS) coefficient σ is set to 1. The labor supply parameter ϕ is set to 1.33, which implies a Frisch elasticity of aggregate hours of 0.75 as in Chetty, Guren, Manoli, and Weber (2011). The parameter that captures the disutility of hours worked ψ is calibrated so that one third of available time is spent at work.

Production. The production function operated by the intermediate goods producers is given by

$$F_t(k_t, l_t) = \left(\alpha_{k,t}(A_{k,t}k_t)^\rho + \alpha_{l,t}(A_{l,t}l_t)^\rho\right)^{1/\rho}. \tag{19}$$

The elasticity of substitution between capital and labor, captured by the parameter ρ, is set to 0.20 as in Karabarbounis and Neiman (2014). The capital- and labor-augmenting technology parameters, A_k and A_l, are normalized to one, without loss of generality. The capital depreciation rate δ is set to 0.072, which is equal to its value leading to 1982 (over the

Table 1: Benchmark Calibration

Parameter	Symbol	Value	Source/Target
Preferences			
Discount factor	β	0.96	Risk-free rate = 4.1%
CEIS parameter	σ	1.00	-
Labor supply elasticity	ϕ	1.33	Chetty et al. (2011)
Disutility of hours worked	ψ	9.65	Labor supply = 1/3
Production			
Elasticity of substitution btwn. capital and labor	ρ	0.20	KN
Depreciation rate	δ	0.072	BEA
Production function parameter	α_k	0.295	Labor share = 0.64 (BLS)
Elasticity of substitution btwn. intermediate inputs	ε	100	Profit share = 0.01 (BB)
Government policy			
Tax rate on labor income	τ_l	29%	MP
Tax rate on capital income	$\tau_k = \tau_\pi$	40%	MP
Government expenditure	g/y	0.20	FRED
Government debt	b/y	0.31	FRED

The table reports the calibration of the model parameters to the early 1980's U.S. economy. The acronyms BB, BEA, BLS, FRED, KN, and MP stand for Barkai and Benzell (2018), Bureau of Economic Analysis, Bureau of Labor Statistics, Federal Reserve Bank of St. Louis FRED database, Karabarbounis and Neiman (2014), and McGrattan and Prescott (2010).

period 1970-1982), calculated from Bureau of Economic Analysis (BEA) National Income and Product Accounts (NIPA) and BEA Fixed Asset Tables (FA). The parameter that governs the elasticity of substitution between intermediate inputs ε is set to 100, implying a profit share of 1% which is equal to the profit share reported by Barkai and Benzell (2018) for the early 1980's, and is also in line with the findings of De Loecker, Eeckhout, and Unger (2020). The production function parameter α_k controls how the remaining 99% income share is divided between capital and labor, and is calibrated to match the observed labor share for the U.S. non-farm business sector over the period of 1947-1982, which we calculate using Bureau of Labor Statistics (BLS) data. The parameter α_l is normalized to $1 - \alpha_k$. The relative price of investment q is normalized to one in the initial steady state.

Government policy. The tax rates for the initial steady state are taken from McGrattan and Prescott (2010). In line with our baseline model, we assume that the tax rate on profit income is equal to the capital income tax rate observed in the data. Accordingly, we set the uniform tax rate on capital and profit income $\tau_k = \tau_\pi$ and labor income τ_l equal to 40% and 29%, respectively. The level of government expenditure is calibrated to match a government expenditure to GDP ratio of 0.20, which is equal to its observed level in the early 1980's calculated by using the St. Louis FED FRED data.

4.2 Calibration: The Evolution of the Economy

There are four time-varying parameters in the model - q_t, $\tau_{k,t}$, ε_t and $\alpha_{k,t}$ (or $1 - \alpha_{l,t}$). In this section, we discuss how they are calibrated. Figure 2 displays the externally calibrated parameters. The series for price of investment, q_t, is computed using St. Louis FED FRED data.[19] The capital tax series, $\tau_{k,t}$, is taken from McGrattan and Prescott (2010). We do not report labor tax rates since, following McGrattan and Prescott (2010), we assume labor tax rate is constant at its initial steady state value over this period. Because we want to abstract from business cycle variations, the capital tax series is smoothed with a piecewise linear function. Similarly, we fit a smooth polynomial form to match the change in price q_t over the period of interest, which captures the fact that (i) the decline in relative price of investment starts in 1983 and (ii) the rate of decline slows down through the end, which implicitly implies that the declining trend in q_t is expected to vanish before 2050.

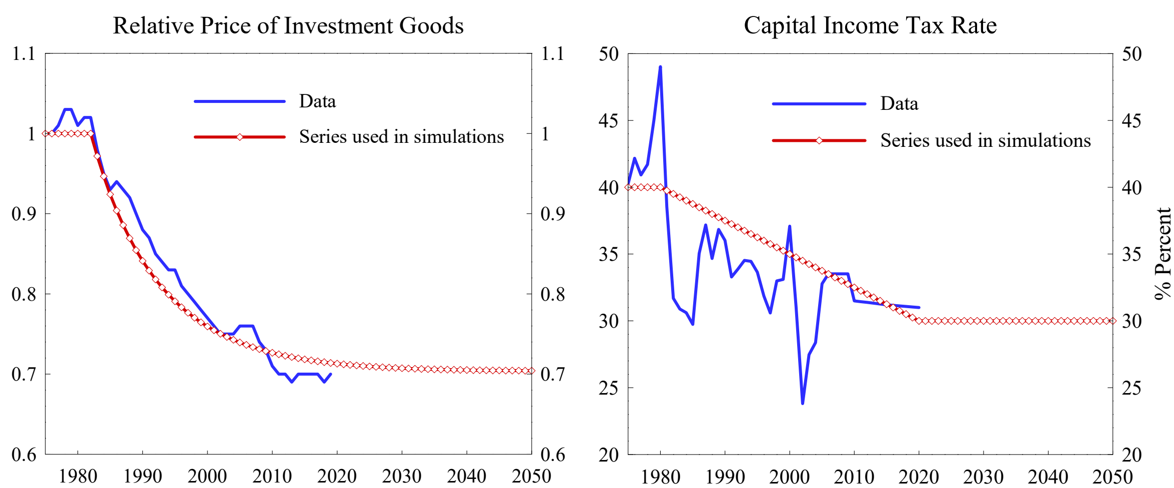


Figure 2: (a) Figure 2: (b)

The figure depicts the relative price of investment goods (a) and the capital income tax rate (b) for the U.S. economy over time.

The remaining two time-varying parameters - ε_t and $\alpha_{k,t}$ - are calibrated internally targeting the empirical evolution of income shares. The evolution of U.S. non-farm labor share is calculated from Bureau of Economic Analysis (BEA) NIPA Tables. The capital share

[19] As in Karabarbounis and Neiman (2014), we construct the relative price of investment series by calculating the ratio of the investment price deflator to the consumption price deflator for the U.S. economy over the post-war period.

series is calculated following the methodology in Karabarbounis and Neiman (2018). The details of labor and capital share computations can be found in Appendix B.1. The profit share, which is the residual, is plotted by the blue line in Figure 3A and displays a rise from a level of 1% to 15% over the period of interest. This is in accordance with findings from notable contributions to the literature on the evolution of US income shares.[20] Recalling that period t profit share in the model equals $1/\varepsilon_t$, ε_t is calibrated to generate a smooth monotonic polynomial fit to the evolution of the empirical profit share. The resulting calibrated ε_t series is depicted with a red line in Figure 3A. The second time-varying input - $\alpha_{k,t}$ - is calibrated to track the observed change in the labor share since the early 1980's, again assuming a smooth monotonic polynomial fit. The empirical and the model implied labor shares (blue and red lines) together with the calibrated $\alpha_{k,t}$ series are depicted in Figure 3B. Both ε_t and $\alpha_{k,t}$ series are imputed after 2020 by extrapolating the polynomial estimates depicted by the red lines.[21] Figure 3 reveals that the model captures the long-term trends in income shares reasonably well.

Model fit. In this section, we investigate how well our calibrated model performs in terms of capturing the evolution of a number of non-targeted key macroeconomic variables. Figure 4A plots the capital-output ratio, and reveals that while the model somewhat undershoots it in the 1980's, it tracks the first decreasing and then stabilizing pattern observed in the data well. Figure 4B and Figure 4C plot two measures of the profitability of private capital (see Farhi and Gourio (2018)), namely, aggregate profits to capital ratio and gross profitability, which is the ratio of aggregate capital income plus profit income to capital. The model broadly captures the observed increasing pattern of profitability as measured by these

[20]For instance, Barkai and Benzell (2018) documents that the profit share increased roughly from a level of 1% to 15% over the period of interest. Barkai (2020) finds that the profit share in the U.S. economy increased roughly by 13.5 percentage points during 1984-2014. According to De Loecker, Eeckhout, and Unger (2020), the profit share for the U.S. economy increased approximately from 2% in the early-1980s to a level of 16% in the late-2010s. Eggertsson, Robbins, and Wold (2018) argue that the profit share, which was roughly zero in the early 1980's, increased to a level of 17% by 2015. Finally, Karabarbounis and Neiman (2018) document that while the labor share and the capital share summed up to 1 in the early 1980's, this number fell down to 0.85 over the period of interest.

[21]More specifically, the time series for ε_t is calibrated specifically to ensure that the profit share i) starts from 1% in 1982, (ii) reaches its long-run level (15%) by the end of 2020's, (iii) matches its actual level in the mid-way of transition in 1999, and (iv) changes following a smooth monotonic polynomial function in time. The decline in $\alpha_{k,t}$ is calibrated to ensure that (i) the simulated labor share equals its empirical counterpart in 1982 and 2016, (ii) the average values of the simulated labor shares for the first and second halves of the period 1983 to 2016 match their data counterparts, and iii) changes in $\alpha_{k,t}$ follows a smooth monotonic polynomial function.

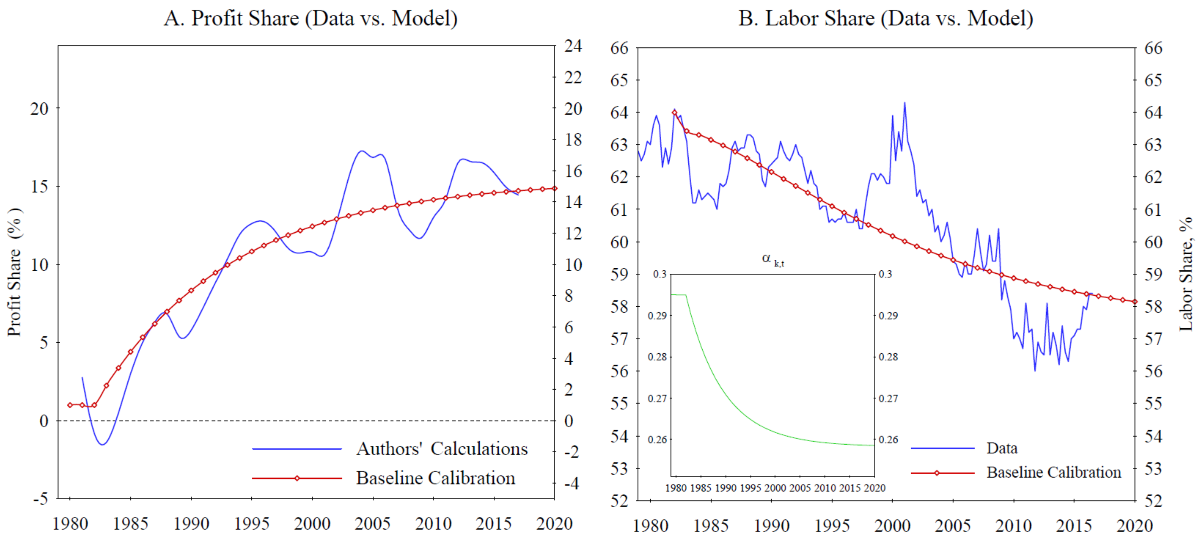


Figure 3: Evolution of income shares

This figure depicts the time series of the observed and the model implied profit share (a) and labor share (b). Panel (b) also plots calibrated $\alpha_{k,t}$ series.

variables. Caballero, Farhi, and Gourinchas (2017), among others, document that a key macroeconomic trend in the US economy observed during the period of interest has been the growing divergence between the return on productive capital and the return on safe assets. More specifically, they show that the difference between the Average Product of Capital (APK) and the return on government bonds (R) has increased significantly since the 1980's. Figure 4D shows that the model captures the initial jump and then the stable, slow rise in $APK - R$ until mid-2000's. Given that the model is too simple to capture important possible factor of the rise in $APK - R$, such as rising macroeconomic risk, it is expected that the model would fall somewhat short of matching the exact rise.[22]

4.3 Optimal Taxes in the Baseline Case

This section reports optimal taxes for the baseline case in which there is a uniform tax rate on capital and profit income and product market policies are not available. We consider two distinct optimal tax reforms. First, the government reforms the tax system now (in year 2021).[23] This exercise, which we refer to as the "2021 reform", aims to inform the policymakers about the following question: given the observed decline in labor share and

[22]The definitions and data sources of the variables considered in this section are given in Appendix B.2.

[23]To be more precise, the economy starts from the early 1980's steady state and the government introduces a one-time, unannounced tax reform in 2021.

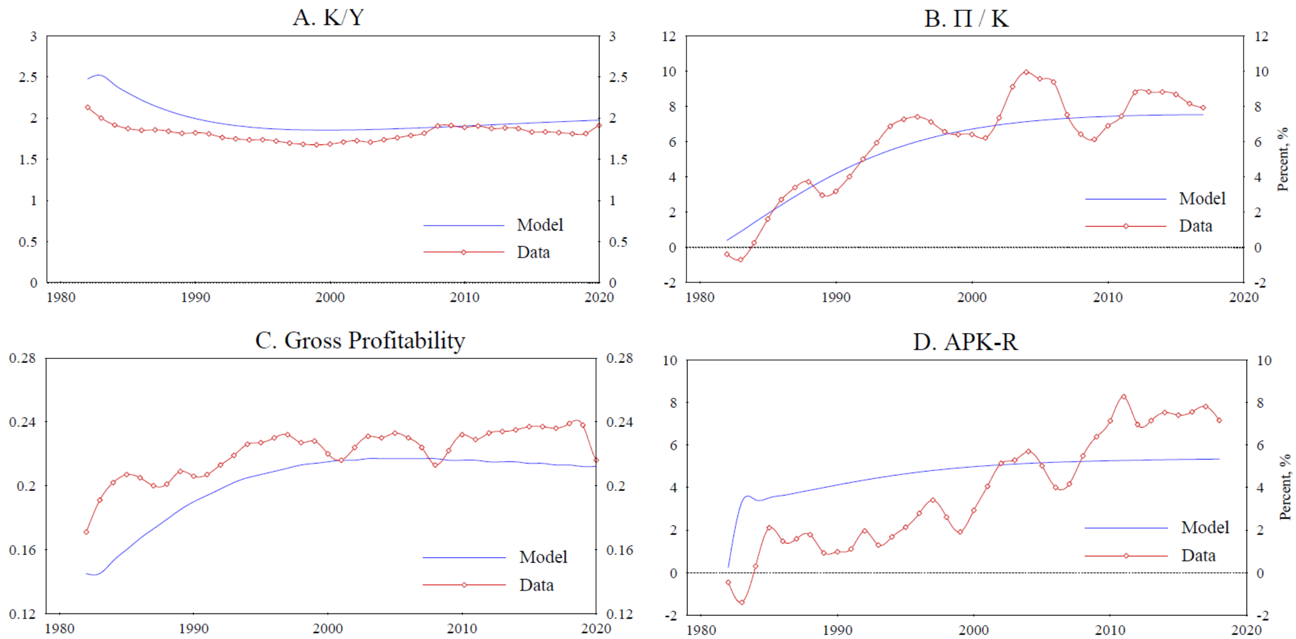


Figure 4: Model fit

This figure depicts the empirical and model implied time series for capital output ratio (a), ratio of profit income to capital stock (b), gross profitability, which is defined as the ratio of capital income plus profit income to capital stock (c) and average product of capital minus real interest rate (d).

the tax policies in place up until now, what is the current optimal policy response? In our second exercise, the tax reform is carried out in 1983. This "1983 reform" informs us about what the policy reaction should have been had the government anticipated the future decline in labor share in 1983. A comparison of the two reforms is informative about the cost of postponing the optimal tax reform. In all exercises, the series of government spending is set exogenously so that it roughly equals 20% of output in the Ramsey allocation in all periods. The initial debt level in both the 1983 and the 2021 reforms are taken from data.[24]

Figure 5A illustrates the time path of optimal capital income taxes. The solid blue line depicts the 2021 reform whereas the red line with diamonds depicts the 1983 reform.[25] In the 2021 reform, the optimal capital tax rate is around its long-run level of 10% throughout

[24]Using the St. Louis FED FRED data, the total federal debt to GDP ratio is calculated to be 0.31 and 1.03 in 1982 and late 2010's, respectively.

[25]As discussed in Section 3, the optimal labor tax rate in period 1 and the optimal tax rate on capital income received in period 2 are qualitatively different from the rest of the policies. We find that period 1 labor tax rate is -17 percent and period 2 capital income tax rate is 566 percent in the 1983 reform while they are -63 percent and 714 percent in 2021 reforms. In order not to dwarf the other tax values in the figure, and in line with the literature, we do not display these initial tax rates on Figure 5 (and later on in figures 8, 9 and 10).

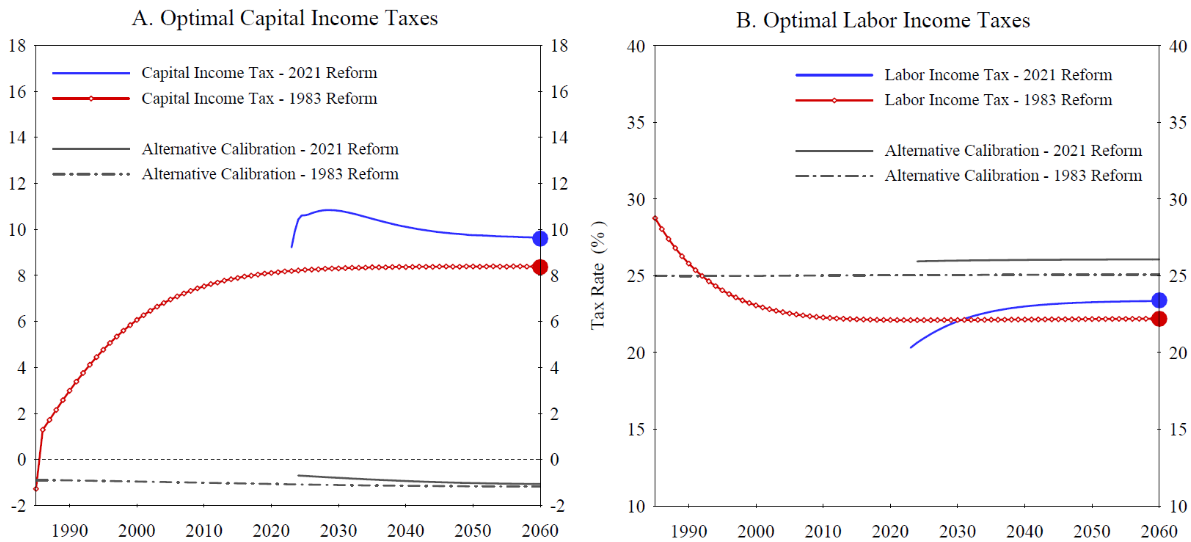


Figure 5: Optimal Tax Rates - Baseline Case

The blue and red lines depict the time series of the optimal capital income tax rates (a) and the optimal labor income tax rates (b) for the baseline case in which there is a uniform tax on profit and capital income and product market policies are not available. The gray solid and dashed lines depict the same for an alternative calibration in which all the decline in labor share is due to an increase in capital share.

the period of interest. In the 1983 reform, the optimal capital tax rate starts from a smaller level around zero and over time it converges to a long-run steady state level of 8%. Figure 5B plots that the optimal labor income tax rate is fairly smooth with a long-run value of 23% in both the 1983 and the 2021 reforms, respectively, which is in line with the standard labor tax smoothing results of Barro (1979) and Lucas and Stokey (1983).

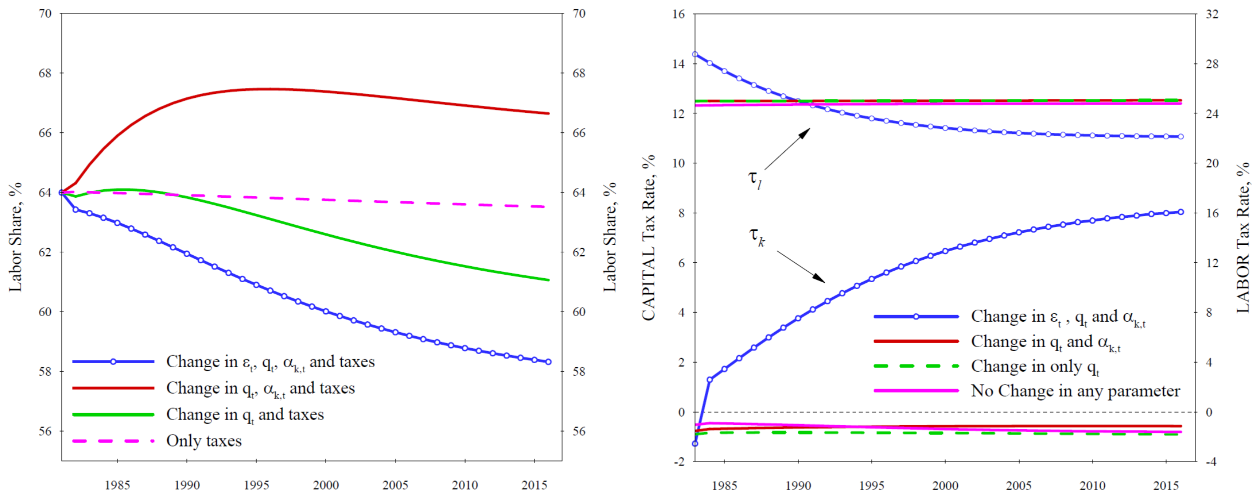
The blue and red lines depicted in Figure 5A have three implications. First, and foremost, in both reforms, the optimal tax rates on capital income are positive and significant which implies that the motives to tax capital coming from the presence of profits are quantitatively significant. Second, the optimal capital tax rate in the 1983 reform starts low and rises with time following the time path of the profit share. This is expected since, as (16) reveals, the strength of both the indirect and the direct profit tax channels depend on the profit share. Third, the optimal long-run capital (and labor) tax rates in the 2021 reform are somewhat larger than those in the 1983 reform even though the long-run profit shares are identical in the two cases. This is because the government has to finance a higher initial debt at the time of the 2021 reform. A glance at the long-run tax formula given by (16) shows that higher revenue requirement for the government imply higher optimal capital taxes via the term χ^*, which represents the relative social value of public funds. One way to interpret this finding is that the cost of delaying the optimal tax reform is forever higher capital income taxes.

Welfare gains. We calculate welfare gains of the optimal tax reforms relative to the equilibrium allocation that arises under the status-quo tax system. The welfare gains of the 1983 reform are equivalent to increasing the consumption of the representative agent by 2.20% at all dates since 1983, while the corresponding welfare gains number is 1.54% for the 2021 reform. This implies that there is considerable welfare loss associated with delaying the optimal tax reform by about four decades.[26]

Alternative calibration: rising capital share. The solid and diamond gray lines in Figure 5A and 5B plot optimal tax rates for an alternative parameterization of the economy in which (i) the parameter ε_t that controls the profit share stays at its early 1980's level and (ii) $\alpha_{k,t}$ series is recalibrated to match the evolution of the observed labor share. This is an economy in which all of the decline in labor share is coming from the rise in capital share. The main finding is that the optimal capital tax rate is zero, which confirms the theoretical results from Section 3: it is optimal to tax capital in our framework in the long run only when there are untaxed profits in the economy.

Impact of each factor in the baseline calibration. The evolution of four exogenous factors, $q_t, \tau_{k,t}, \varepsilon_t, \alpha_{k,t}$, jointly determine the evolution of labor share depicted in Figure 3B. This section quantifies the contribution of the change in each factor on the labor share and disentangles the impact of each on optimal taxes. To do so, we compute the evolution of the labor share in equilibrium for a series of counterfactual economies, which are plotted in Figure 6A. The labor share given by the pink dashed line corresponds to an economy in which only $\tau_{k,t}$ changes over time (as it does in the status-quo system, see Figure 2B) and shows that changes in taxes have virtually no impact on the labor share in our model. The green line corresponds to the labor share of an economy in which $\tau_{k,t}$ and q_t change whereas the red line adds the change in $\alpha_{k,t}$ on top of the changes in $\tau_{k,t}$ and q_t. The green line reveals that the decline in capital prices is a major force for declining labor share, which is to be expected given that under our parameterization of $\rho = 0.2$, capital and labor are gross

[26] Alternatively, one can calculate welfare gains of reforms from 2021 perspective, that is by comparing the allocations that arise under status-quo and each reform from 2021 onward. In this case, 1983 reform provides much larger gains 5.39% vs. 1.95%. This is because the reforms result in costly capital accumulation in the short run and evaluating the 1983 reform from 2021 perspective does not take these costs into account.



(a) Labor Share

(b) Optimal Taxes

Figure 6: Impact of different factors

Panel (a) plots the evolution of the labor share in the model equilibrium for a series of counterfactual economies in which we add a change in a time-varying factor at a time starting with the economy in which only taxes change as in the status-quo system. Panel (b) plots optimal capital and labor income tax rates for a series of counterfactual economies in which we add a change in a time-varying factor at a time starting with the economy in which no factor changes. Here, there is no only taxes exercise since taxes are chosen optimally.

substitutes in our model. The decline in $\alpha_{k,t}$, calibrated to track the trend in labor share, increases labor's share substantially as shown by the red line. Finally, the comparison of the blue line, which redraws the model implied labor share from Figure 3B, with the red line indicates that the rise in ε_t is the largest determinant of the labor share.

Figure 6B plots the impact of the evolution of each factor on optimal taxes, only for the 1983 reform for brevity. The pink dashed, red and green lines depict optimal tax rates for economies in which there is no change in exogenous factors, only q_t changes, q_t and $\alpha_{k,t}$ changes, and all the parameters change. The figure reveals that, in the absence of a change in ε_t, changes in other parameters are not consequential for optimal tax rates.[27] We also compute welfare gains of switching from status-quo taxes to optimal taxes for each counterfactual economy. For the no change economy, the welfare gains of the switch to optimal taxes is significant at 2.4%. The importance of the optimal tax reform is larger at 3.5% when the decline in capital prices are taken into account. Intuitively, the suboptimally high status-quo

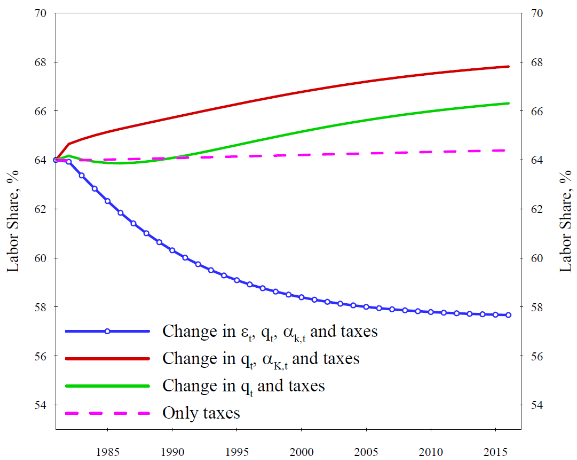
[27]Due to the presence of significant non-linearities in the model, the order in which the evolution of time-varying factors is added matters for the labor share and the optimal taxes. We follow the order in which evolution of ε_t is added last since it facilitates highlighting the insignificance of other factors in the absence of large profit shares.

capital tax rates hurt the economy more in the more capital-intensive economy that results when capital prices fall. Likewise, the welfare gains of the optimal reform falls from 3.5% to 1.6% when we add the changes in $\alpha_{k,t}$ over time since this reduces the capital intensity of production. The welfare gains increase considerably from 1.6% to 2.2% when taking into account the rise in ε_t, partly because taxes are now also used to correct inefficiencies.

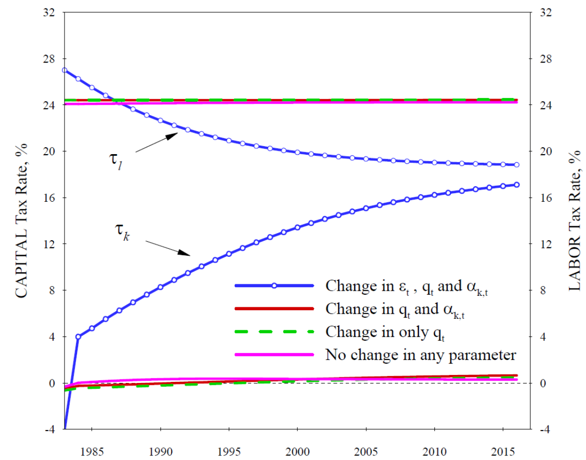
Sensitivity to ρ. In our benchmark calibration, following Karabarbounis and Neiman (2014), we assume $\rho = 0.2$, which implies capital and labor are gross substitutes in production. There is, however, a wide range of estimates for this key parameter. (See Rognlie (2015) Section 3A for a discussion.) We conduct a sensitivity exercise in which we set $\rho = -0.25$, (Oberfield and Raval (2021)) under which capital and labor are gross complements. Keeping the time series of all the other factors the same as in the $\rho = 0.2$ calibration, we recalibrate $\alpha_{k,t}$ to match the observed trend in the labor share given by Figure 3B. Figure 7A plots the contribution of the four time-varying factors to the change in labor share. The main difference to the $\rho = 0.2$ case is that the fall in q_t raises the labor share when $\rho = -0.25$. The change in $\alpha_{k,t}$ still increases the labor share, but to a lesser extent. Figure 7B displays that the rise in the optimal capital tax rate is significantly larger relative to the gross substitutability case. The optimal long-run rate is 18% when $\rho = -0.25$ vs. 8% in the $\rho = 0.2$ case. Numerical evaluation of the terms in optimal tax formula (16) at corresponding Ramsey allocations offers an explanation. All the terms in the formula except for the elasticity term, $\mathcal{E}^*_{1-\tau_k,k}$, are very similar across the two economies. $\mathcal{E}^*_{1-\tau_k,k}$ is lower in $\rho = 0.2$ case, implying that capital accumulation is more sensitive to a rise in the capital tax rate, which implies that rising the capital tax creates a larger deadweight loss, calling for lower optimal tax rate. Intuitively, higher degree of substitutability between capital and labor implies higher sensitivity of capital to taxation. The fact that $\tilde{S}^*_k$ and χ^* are similar across the two economies follows from (i) the recalibration of $\alpha_{k,t}$ to ensure capital shares are comparable, (ii) government spending to GDP ratio being the same across the two Ramsey problems.

4.4 Optimal Taxes with Product Market Policies

In this section, we display optimal taxes for the case in which there is a uniform tax on capital and profit income and government has access to product market policies. Figure 8A illustrates the path of optimal capital income taxes, and reveals that the qualitative pattern



(a) Labor Share



(b) Optimal Capital Tax

Figure 7: Sensitivity to ρ

This figure plots labor share and optimal taxes for the case in which $\rho = -0.25$. Panel (a) plots the evolution of the labor share in the model equilibrium for a series of counterfactual economies in which we add a change in a time-varying factor at a time starting with the economy in which only taxes change as in the status-quo system. Panel (b) plots optimal capital and labor income tax rates for a series of counterfactual economies in which we add a change in a time-varying factor at a time starting with the economy in which no factor changes. Here, there is no only taxes exercise since taxes are chosen optimally.

of optimal taxes over time in both 1983 and 2021 reforms are similar to those in the baseline case without product market policies (Figure 5A). The magnitudes, however, differ: optimal capital tax rates are much higher in the presence of product market policies. A comparison of the long-run optimal capital tax formula in the absence of product market policies, given by (16), with the formula for the case with product market policies, given by (17), reveals the reason. In the former case, there is an additional term, -1, in the parenthesis that calls for a subsidy on capital income. This term is absent in the implementation with product market policies since in this case monopolistic distortions are dealt with at the firm level where they originate. Figure 8B shows that, in line with standard labor tax smoothing results, the optimal tax rate on labor income is roughly constant in both 1983 and 2021 reforms.

4.5 Optimal Taxes under Exogenous Profit Taxes

In the case with exogenous profit taxes, the sequence of tax rates on profits is an additional parameter in the Ramsey planning problem. Following the calibration in Section 4.2, we set the profit income tax sequence to the sequence of status-quo capital income taxes.[28] Figure

[28]The profit tax rate decreases from 40% to 30% following the observed capital income tax series in the calibration of Section 4.2. As a sensitivity analysis, we also consider an alternative calibration of the profit

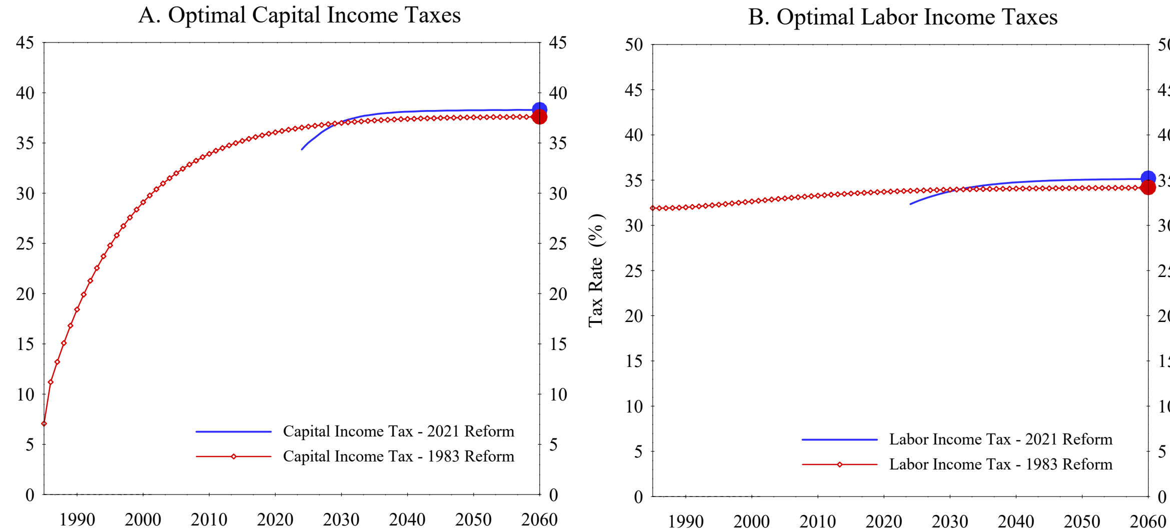


Figure 8: Optimal Tax Rates with Product Market Policies

This figure depicts the time series of the optimal capital income tax rates (a) and the optimal labor income tax rates (b) for the case in which there is a uniform tax that applies to capital and profit income and product market policies are available.

9A, which displays the time path of optimal capital income taxes, indicates that the optimal capital taxes are positive and significant under both 1983 and 2021 reforms. Moreover, the optimal capital tax rates are smaller in comparison to those in the case where capital and profit income are taxed at the same rate, given by Figure 8A. A comparison of the optimal long-run tax formulas (17) and (18) in the previous section sheds light on why: when capital and profit taxes are set separately, there is no direct profit tax revenue benefit - represented by $\chi^* \mathcal{E}^*_{1-\tau_k,k}/\tilde{S}^*_k$ in (17) - coming from raising the capital tax rate.[29]

5 Optimal Taxation in an Inefficient Economy

The optimal tax analysis so far has been carried out under the assumption that the government corrects monopolistic distortions that slow down capital accumulation and cause underemployment, be it via subsidizing capital and labor income or via product market

tax sequence in which it is set to equal the observed tax rate on distributions (see McGrattan and Prescott 2010), decreasing from a level of 40% to 15% over the period of interest. We find that the optimal capital income taxes are again positive, and somewhat higher than the ones implied by the benchmark calibration. The details of this alternative calibration procedure and the optimal tax results are given in Appendix C.

[29]Looking at Figure 9A, one may wonder as to why the optimal capital tax rate is negative in the first few periods following the 1983 reform. A detailed discussion of why it may be optimal to subsidize capital income in the short run in the model with exogenous profit taxes and why this never occurs in the long run is provided in Appendix A.5.

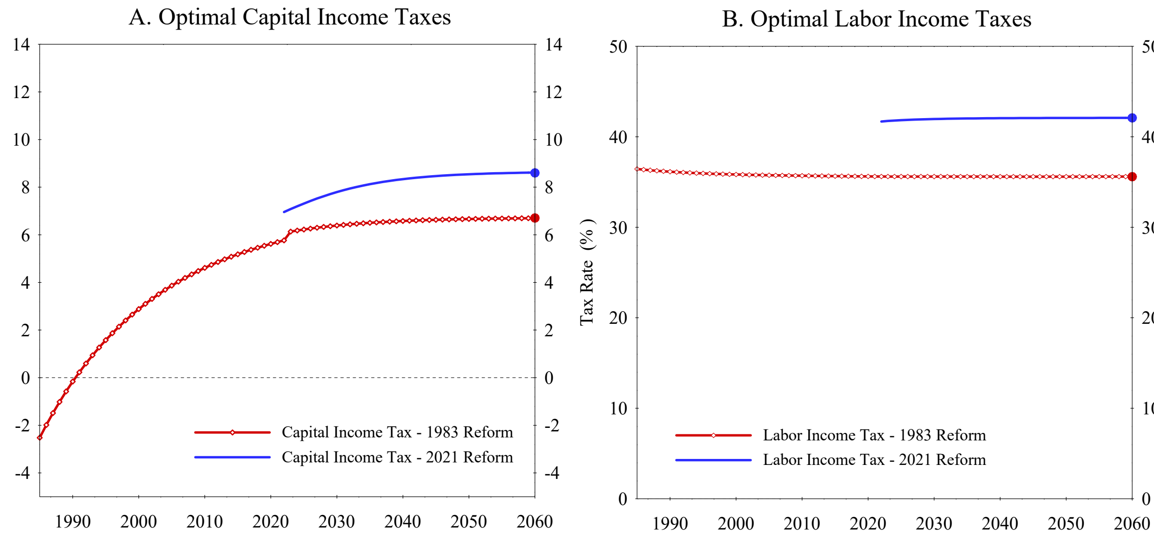


Figure 9: Optimal Tax Rates under Exogenous Profit Taxes

This figure depicts the time series of the optimal capital income tax rates (a) and the optimal labor income tax rates (b) for the case in which profits are taxed separately from capital income at an exogenous rate and product market policies are available.

policies. However, in reality, structural problems such as product market distortions might be hard to solve for various reasons.[30] Alternatively, one may think that correcting product market distortions is the concern of other regulatory bodies within the government, and an all-encompassing reform that includes changing tax rates and regulating the product market distortions at the same time might be hard to achieve. In either case, there is a motive to analyze an optimal tax reform in a world in which product market distortions are not corrected. This section addresses this issue by analyzing optimal Ramsey taxation in a world with inefficiently low levels of capital accumulation and employment resulting from monopolistic distortions. To do so, we modify the Ramsey problem given by (14) as follows.

Ramsey problem. Given (k_1, b_1), initial policies $\tau_{\pi,1} = \tau_{k,1} = \bar{\tau}_{k,1}$, a sequence $\{\tilde{\pi}_t\}_{t=1}^{\infty}$ and a sequence of government spending $\{g_t\}_{t=1}^{\infty}$, government chooses allocation (c, k, l) to solve the following problem:

[30] One such reason, for instance, might be that the degree of monopolistic distortions are heterogenous across firms and depend on unobservable firm characteristics. In such a world, it would be difficult to fully correct monopolistic distortions as the magnitude of Pigouvian corrections would depend on characteristics privately observed by firms. See Boar and Midrigan (2020) for an analysis of optimal regulation of monopolistic distortions in the presence of informational frictions.

$$\max_{c,k,l} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t) \qquad \text{s.t.} \tag{20}$$

$$c_t + q_t k_{t+1} + g_t \leq \left(1 - \frac{1}{\varepsilon_t}\right) F_t(k_t, l_t) + \tilde{\pi}_t + (1-\delta) q_t k_t, \quad \text{for all } t,$$

$$\sum_{t=1}^{\infty} \beta^{t-1} \left(u_{c,t} c_t + u_{l,t} l_t\right) = \sum_{t=1}^{\infty} \beta^{t-1} u_{c,t} (1 - \tau_{\pi,t}) \pi_t + u_{c,1}(\bar{r}_1 k_1 + b_1),$$

where $\pi_t = \frac{1}{\varepsilon_t} F_t(k_t, l_t)$, $\tau_{\pi,t}$ is given by (13), and in each period t, exogenously given profit income coincides with the profit income that the Ramsey problem generates: $\tilde{\pi}_t = \frac{1}{\varepsilon_t} F_t(k_t, l_t)$.[31]

To understand this problem, recall that in the laissez-faire equilibrium of our growth model, the private marginal returns to capital and labor faced by the intermediate goods producers are $\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t}$ and $\left(1 - \frac{1}{\varepsilon_t}\right) F_{l,t}$, respectively. The social marginal returns in the Ramsey planning problem given by (14), on the other hand, equal $F_{k,t}$ and $F_{l,t}$. Technically, it is these wedges between private and social returns that make it optimal to subsidize capital and labor (be it implemented at the consumer or firm level). The planning problem (20) modifies the planning problem (14) to ensure that the marginal returns to capital and labor perceived by the Ramsey planner are equal to the private returns firms face in equilibrium. This is achieved by revising the resource constraint as in (20) by writing output as a summation of two parts: $\left(1 - \frac{1}{\varepsilon_t}\right) F(k_t, l_t)$ and $\tilde{\pi}_t$. The first part ensures that the Ramsey planner perceives the same returns as the private agents. By setting the second part to $\tilde{\pi}_t = \frac{1}{\varepsilon_t} F(k_t, l_t)$, we ensure that, as in the market equilibrium, total output equals $F_t(k_t, l_t)$. This formulation guarantees that monopolistic distortions are not corrected in the solution to the Ramsey problem (20). The first-order optimality conditions of (20) provided in Appendix A.6, together with (8)-(9) evaluated at the steady state, deliver Proposition 6.

Proposition 6. *The long-run optimal tax rate on capital income is given by*

$$\frac{\tau_k^*}{1 - \tau_k^*} = \frac{S_\pi F_k^*}{\left(1 - \frac{1}{\varepsilon}\right) F_k^* - \delta q} \chi^* \left[1 + \mathcal{E}^*_{1-\tau_k, k} / \tilde{S}_k^*\right]. \tag{21}$$

Proof. Relegated to Appendix A.6. □

Notice that the optimal capital tax formula given by (21) is quite similar to the optimal

[31] The constraint that $\tau_{\pi,t}$ is given by (13) in the Ramsey problem (20) shows that the optimal tax analysis for the inefficient economy is carried out under the assumption of uniform taxation of capital and profit income. Our main results are valid in the alternative case with exogenous profit taxes. This case is not presented here for the sake of brevity, and is available upon request.

capital tax formula for the case in which distortions are corrected via product market policies given by (17). Specifically, the capital subsidy term, -1, which is present in the optimal capital tax formula in the case without product market distortions, (16), is absent from both the optimal tax formulas (17) and (21). The reason for why this term is absent differs across the two environments, however. While product market interventions eliminate the distortions and, therefore, the need for a subsidy on capital income in (17), the assumption that the Ramsey planner does not correct monopolistic distortions imply that there is no such motive for a subsidy in the economy that corresponds to (21). It is only the first terms, $\frac{1}{F_k^* - \delta q}$ vs. $\frac{1}{(1-\frac{1}{\varepsilon})F_k^* - \delta q}$, that differ across the capital tax formulas (17) and (21). This reflects the fact that the interest rate, and hence the tax base, is inefficiently low in the inefficient economy due to monopolistic distortions.

5.1 Quantitative Analysis

Figure 10A plots the path of optimal capital income taxes for the inefficient economy, and reveals that the qualitative pattern of optimal taxes over time for both 1983 and 2021 reforms are similar to those in cases in which monopolistic distortions are corrected (via consumer level subsidies, Figure 5A or product market policies, Figure 8A). Comparing Figure 8A and Figure 10A, we see that the optimal taxes in the inefficient economy are lower compared to the case where monopolistic distortions are corrected with product market subsidies. This may be surprising given that the tax base in the inefficient economy, $\frac{1}{(1-\frac{1}{\varepsilon})F_k^* - \delta q}$, is smaller than that in the economy with corrective subsidies, $\frac{1}{F_k^* - \delta q}$, which implies that, all else equal, a comparison of the optimal tax formulas delivers a higher capital tax rate for the inefficient economy. The opposite is true because all else is not equal across the two cases: namely, the relative social value of public funds, χ^*, is smaller in the inefficient economy. χ^* is lower for the inefficient economy because, while the the social value of public funds, measured by λ^*, is the same across the two Ramsey problems - (14) and (20) - (since government spending to GDP ratios are aligned) the social cost of distorting capital accumulation, μ^*, is higher in the solution to (20). This is because the level of capital (and labor) is suboptimal to begin with in the inefficient economy due to the presence of monopolistic distortions.

Unlike the cases in which monopolistic distortions are corrected, capital taxes are higher in the 1983 reform relative to the 2021 reform in the inefficient economy. This happens because, χ^*, which measures the social value of public funds relative to the social cost of distorting capital accumulation, is higher in the 1983 reform despite the fact that initial public debt, and hence, the social value of public funds, measured by the multiplier on the implementability constraint, λ^*, is larger in the 2021 reform. This is because the social cost of distorting capital accumulation, measured by the multiplier on feasibility constraint, μ^*, is higher in the 2021 reform since in this reform the economy starts already with a larger degree of (uncorrected) monopolistic distortions whereas the 1983 reform experiences a transition with a few initial decades of low distortions.[32]

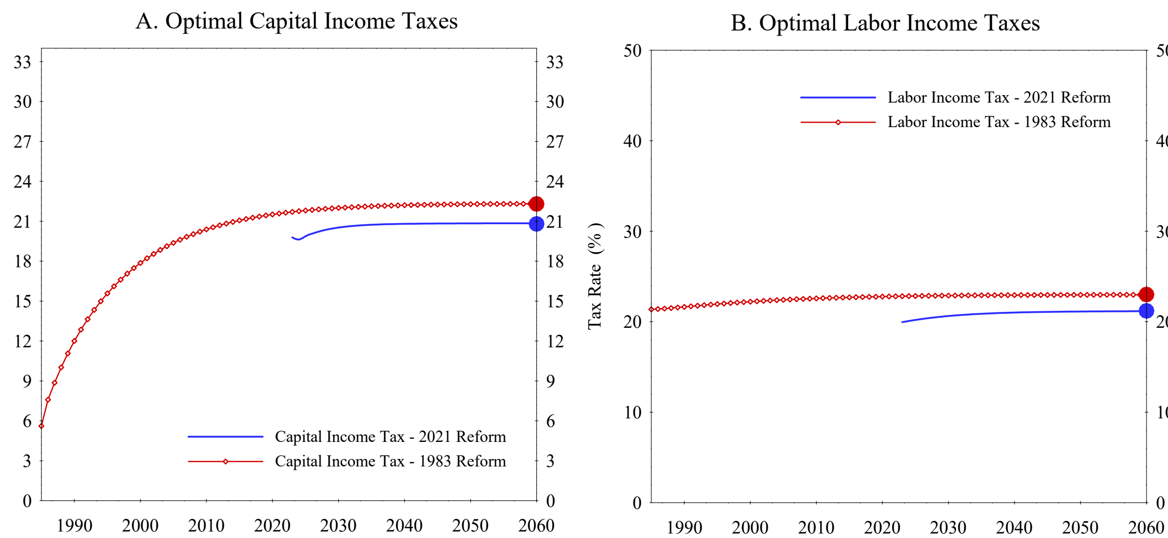


Figure 10: Optimal Tax Rates in the Inefficient Economy

This figure depicts the time series of the optimal capital income tax rates (a) and the optimal labor income tax rates (b) for the economy in which government cannot correct monopolistic distortions.

6 Conclusion

Numerous recent studies have documented that the labor's share in national income has been declining at a considerable rate since the early 1980's. In this paper, we analyze the implications of this decline for tax policy from the perspective of a government that needs to finance spending. We find that the optimal tax implications of the decline in the labor share depend on the mechanism responsible for it. In particular, if the labor share has declined due to a decline in competition or other mechanisms that raise the share of profits in national

[32]Technically, under our calibration which implies rising market power between 1983 and 2021, the aggregate production function following the 2021 reform is inferior relative to the aggregate production function following the 1983 reform for roughly the first three decades. This is why μ^* is relatively larger in the 2021 reform.

income, then it should optimally be accompanied with a rise in capital income taxes. If, on the other hand, the labor share has declined because of the rise in automation or other mechanisms that make the production more capital intensive, then it has no bearing on optimal capital income taxation. A quantitative application shows that soaring profit shares since the 1980's can justify significant tax hikes on capital income for the U.S. economy.

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Appendix - For Online Publication

A Proofs

A.1 Proof of Proposition 1

Proof. We first show that equilibrium allocation satisfies (11), (12), and (13). The fact that equilibrium allocation satisfies (11) follows from the fact that it satisfies (4) and (10) with equality.

To see (12), first notice that if we plug (6) into (4) and use the transversality condition $\lim_{t\to\infty} p_t q_t k_{t+1} = 0$, we achieve

$$\sum_{t=1}^{\infty} p_t c_t = \sum_{t=1}^{\infty} p_t \left(w_t l_t (1-\tau_{l,t}) + \pi_t (1-\tau_{\pi,t}) \right) + p_1 b_1, \tag{22}$$

Normalizing $p_1 = 1$, plugging in (5) and (7) into (22) and, multiplying both sides by $u_{c,1}$, we prove that the allocation satisfies (12).

When we combine the first-order optimality conditions of the consumer, (5) and (6), with the equilibrium rental rate of capital given by (3), we see that in equilibrium:

$$u_{c,t-1} q_{t-1} = \beta u_{c,t} \left[q_t + \left(\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t \right) (1-\tau_{k,t}) \right]. \tag{23}$$

Deriving $1-\tau_{k,t}$ from (23) and recalling $\tau_{\pi,t} = \tau_{k,t}$ proves that equilibrium allocation satisfies (13).

Now, we prove the other direction. Suppose an allocation together with initial policies satisfies (11), (12), and (13). We will show that this allocation, with properly constructed prices and taxes, constitutes a tax-distorted equilibrium. First, use (3) and $\pi_t = \frac{1}{\varepsilon_t} y_t$ to construct factor prices and profit income every period. Normalize $p_1 = 1$ and use (5) to set p_t. Use (23) to construct capital (and profit) income taxes for periods $t \geq 2$ and (7) to construct labor income taxes for periods $t \geq 1$. Given this construction, the allocation satisfies consumer and firm optimality conditions. Using the constructed prices and taxes and the transversality condition in (12), we obtain that the allocation satisfies consumer budget constraint (4). Combining (4) with the resource constraint (11) gives the government budget constraint (10), which completes the proof. $\square$

A.2 Proof of Proposition 2

Proof. We first provide the complete set of the first-order optimality conditions of (14).

First-order optimality conditions of Ramsey Problem (14). Although the first-order optimality conditions for capital and labor are provided in the main text, we report them here again for completeness. Letting $\beta^{t-1}\mu_t$ and λ be LaGrange multipliers on period t feasibility constraint and implementability constraint, the full set of first-order conditions are as follows. For $t \geq 2$:

$$\begin{aligned}(k_t) : -\beta^{t-2}\mu_{t-1}^* q_{t-1} + &\beta^{t-1}\mu_t^* \left(F_{k,t}^* + (1-\delta)q_t\right) \\ &-\lambda^*\beta^{t-1}u_{c,t}^*\left[(1-\tau_{\pi,t}^*)\frac{\partial \pi_t^*}{\partial k_t} + \frac{\partial(1-\tau_{\pi,t}^*)}{\partial k_t}\pi_t^*\right] = 0, \end{aligned} \tag{24}$$

$$\begin{aligned}(l_t) : \beta^{t-1}u_{l,t}^* + &\lambda^*\beta^{t-1}\left[u_{ll,t}^* l_t + u_{l,t}^*\right] \\ &-\lambda^*\beta^{t-1}u_{c,t}^*\left[(1-\tau_{\pi,t}^*)\frac{\partial \pi_t^*}{\partial l_t} + \frac{\partial(1-\tau_{\pi,t}^*)}{\partial l_t}\pi_t^*\right] + \beta^{t-1}\mu_t^* F_{l,t}^* = 0, \end{aligned} \tag{25}$$

$$\begin{aligned}(c_t) : \beta^{t-1}u_{c,t}^* &+ \lambda^*\beta^{t-1}\left[u_{cc,t}^* c_t^* + u_{c,t}^*\right] - \lambda^*\beta^{t-1}u_{cc,t}^*(1-\tau_{\pi,t}^*)\pi_t^* \\ &- \lambda^*\beta^{t-1}\left[u_{c,t}^*\frac{\partial(1-\tau_{\pi,t}^*)}{\partial c_t}\pi_t^* + \beta u_{c,t+1}^*\frac{\partial(1-\tau_{\pi,t+1}^*)}{\partial c_t}\pi_{t+1}^*\right] - \beta^{t-1}\mu_t^* = 0. \end{aligned} \tag{26}$$

The first-order optimality conditions for consumption and labor are different for $t = 1$:

$$\begin{aligned}(c_1) :\quad & u_{c,1}^* + \lambda^*\left[u_{cc,1}^* c_1^* + u_{c,1}^* - u_{cc,1}^*(1-\tau_{\pi,1}^*)\pi_1^* - \beta u_{cc,2}^*\frac{\partial(1-\tau_{\pi,2}^*)}{\partial c_1}\pi_2^* - u_{cc,1}A_1\right] - \mu_1^* = 0, \\ (l_1) :\quad & u_{l,1}^* + \lambda^*\left[u_{ll,1}^* l_1^* + u_{l,1}^* - u_{c,1}^*(1-\bar{\tau}_{\pi,1})\frac{\partial \pi_1^*}{\partial l_1}\right] + \mu_1^* F_{l,1}^* = 0,\end{aligned}$$

where $A_1 = \bar{r}_1 k_1 + b_1$ is the real value of initial assets.

At the steady state (24) becomes

$$(k) : -\mu^* q + \mu^*\left(F_k^* + (1-\delta)q\right) - \lambda^* u_c^*\left[(1-\tau_\pi^*)\frac{\partial \pi^*}{\partial k} + \frac{\partial(1-\tau_\pi^*)}{\partial k}\pi^*\right] = 0. \tag{27}$$

Combining (27) with the steady-state version of (8), and rearranging gives:

$$\tau_k^* = \frac{1}{\left(1-\frac{1}{\varepsilon}\right)F_k^* - \delta q}\left(-\frac{1}{\varepsilon}F_k^* + \chi^*\left[\frac{\partial \pi^*}{\partial k}(1-\tau_\pi^*) + \frac{\partial(1-\tau_\pi^*)}{\partial k}\pi^*\right]\right), \tag{28}$$

where $\chi^* = \frac{\lambda^* u_c^*}{\mu^*}$.

By taking the right-hand side of (28) into $(1-\tau_\pi^*)$ parenthesis and using $\tau_\pi^* = \tau_k^*$, $\pi^* = S_\pi Y^*$, and $\frac{\partial \pi^*}{\partial k} = S_\pi F_k^*$, we obtain

$$\frac{\tau_k^*}{1-\tau_k^*} = \frac{F_k^*}{(1-\frac{1}{\varepsilon})F_k^* - \delta q}S_\pi\left[-\frac{1}{1-\tau_k^*} + \chi^*\left(1+\mathcal{E}^*_{1-\tau_k,k}/\tilde{S}_k^*\right)\right], \tag{29}$$

where $\mathcal{E}^*_{1-\tau_k,k} = \frac{d\ln(1-\tau_k)}{dk}|_{k=k^*}$ is the elasticity of the retention rate on capital income with respect to the equilibrium level of capital stock and $\tilde{S}_k^* = \frac{F_k^* k^*}{Y^*}$ is the inverse of capital's share in production. Solving for $\frac{\tau_k^*}{1-\tau_k^*}$ delivers the capital tax formula (16) in Proposition 2.

□

Steady-state optimal labor tax formula. Suppose $\tau_{k,t} = \tau_{\pi,t}$ for all $t \geq 1$. The long-run optimal tax rate on labor income is given by

$$\tau_l^* = 1 - \frac{1+\lambda^*\left(1+u_{cc}^* c^*/u_c^*\right)}{1+\lambda^*\left(1+u_{ll}^* l^*/u_l^*\right)}\left(1-\chi^* S_\pi(1-\tau_k^*)\left[1+\mathcal{E}^*_{1-\tau_k,l}/\tilde{S}_l^*\right]\right)\frac{1}{1-S_\pi}, \tag{30}$$

where $\mathcal{E}^*_{1-\tau_k,l} = \frac{d\ln(1-\tau_k)}{dl}|_{l=l^*}$ is the elasticity of the retention rate on capital income with respect to the equilibrium level of labor and $\tilde{S}_l^* = \frac{F_l^* l^*}{Y^*}$ is the inverse of labor's share in production.

Proof. At the steady state (25) and (26) become

$$(l) : u_l^* + \lambda^*\left[u_{ll}^* l^* + u_l^*\right] - \lambda^* u_c^*\left[(1-\tau_\pi^*)\frac{\partial \pi^*}{\partial l} + \frac{\partial(1-\tau_\pi^*)}{\partial l}\pi^*\right] + \mu^* F_l^* = 0 \tag{31}$$

and

$$(c) : u_c^* + \lambda^*\left[u_{cc}^* c^* + u_c^*\right] - \mu^* = 0. \tag{32}$$

Combining (31) and (32) with the steady-state version of (9), and rearranging gives the labor tax formula (30). □

The term $\frac{1+\lambda^*(1+u^*_{cc}c^*/u^*_c)}{1+\lambda^*\left(1+u^*_{ll}l^*/u^*_l\right)}$ is the standard Ramsey optimal steady-state labor tax formula whereas the terms that follow appear due to presence of untaxed profits.

A.3 Proof of Proposition 3

Proof. For brevity, here we only prove the proposition for the baseline case in which there is a uniform tax on capital and profit income and there are no product market subsidies. The proofs for the other two cases are very similar.

Suppose $S_{\pi,t-1} = S_{\pi,t} = S_{\pi,t+1} = 0$ for some $t \geq 3$. That $S_{\pi,t} = 0$ implies that the first-order condition for capital given by (24) becomes

$$(k_t) : -\mu^*_{t-1}q_{t-1} + \mu^*_t \left(F^*_{k,t} + (1-\delta)q_t\right) = 0. \tag{33}$$

Together with the assumption on preferences, that $S_{\pi,t} = S_{\pi,t+1} = 0$ implies that the first-order optimality condition for consumption for period t given by (26) becomes

$$(c_t) : \beta^{t-1}u^*_{c,t}\left[1 + \lambda^*(1-\sigma)\right] - \mu^*_t = 0. \tag{34}$$

That $S_{\pi,t-1} = S_{\pi,t} = 0$ implies that the first-order optimality condition for consumption for period $t-1$ becomes

$$(c_{t-1}) : \beta^{t-2}u^*_{c,t-1}\left[1 + \lambda^*(1-\sigma)\right] - \mu^*_{t-1} = 0. \tag{35}$$

Combining (33), (34) and (35), we get

$$u^*_{c,t-1}q_{t-1} = \beta u^*_{c,t}\left(F^*_{k,t} + (1-\delta)q_t\right). \tag{36}$$

Plugging (36) into (8) evaluated at $\varepsilon_t^{-1} = 0$ gives $\tau^*_{k,t} = 0$. That $S_{\pi,t} = 0$ implies that the first-order condition for labor given by (25) becomes

$$(l_t) : \beta^{t-1}u^*_{l,t} + \lambda^*\beta^{t-1}\left[u^*_{ll,t}l_t + u^*_{l,t}\right] + \mu^*_t F^*_{l,t} = 0. \tag{37}$$

Plugging (34) and (37) into (9) evaluated under $\varepsilon_t^{-1} = 0$ gives the result. $\square$

A.4 Implementation with Product Market Policies

The product market policies we consider are of the form: for all t,

$$\hat{\tau}_{s,t} = \frac{1}{\varepsilon_t - 1},$$

$$\hat{T}_t = y_t \left(\frac{1}{\varepsilon_t - 1} - \frac{1}{\varepsilon_t} \frac{F_{k,t} - \delta q_t}{\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t} \right). \tag{38}$$

In this appendix, we show that under these product market policies (i) the equilibrium rental rates on capital and labor equal their marginal products; (ii) the set of equilibrium allocations that are attainable without product market policies is equivalent to the set of equilibrium allocations that are attainable with them. The latter implies that introducing product market policies do not alter the set of allocations available to the government, and hence, the Ramsey Problem given by (14) still characterizes this set under product market policies.

Before establishing these claims, we first describe how the existence of the product market policies described by (38) affects market equilibrium. An intermediate good producer's problem becomes:

$$\max_{\xi_{i,t}} (1 + \hat{\tau}_{s,t}) \xi_{i,t} y_{i,t} - m_{i,t} y_{i,t} - \hat{T}_t$$

subject to the demand for that intermediate good. The presence of product market policies also alter the government's budget constraint as follows:

$$\sum_{t=1}^{\infty} p_t \left(g_t + \tau_{s,t} y_t \right) + p_1 b_1 = \sum_{t=1}^{\infty} p_t \left(w_t l_t \tau_{l,t} + (r_t - q_t \delta) k_t \tau_{k,t} + \pi_t \tau_{\pi,t} + T_t \right). \tag{39}$$

The definition of equilibrium with product market policies is identical to the definition of market equilibrium given in Section 2 except that the intermediate goods producers' problem and the government budget constraint are modified as above.

The following proposition establishes claims (i) and (ii).

Proposition 7. *Given (k_1, b_1) and $\{g_t\}_{t=1}^{\infty}$, suppose the allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$, together with prices $\{p_t, r_t, w_t\}_{t=1}^{\infty}$, profits $\{\pi_t\}_{t=1}^{\infty}$, and taxes $\{\tau_{k,t}, \tau_{l,t}, \tau_{\pi,t}\}_{t=1}^{\infty}$ constitute a tax-distorted market equilibrium without product market policies. Then, $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ is also an equi-*

librium allocation under product market policy given by (38) with appropriately constructed prices and taxes. Moreover, in this equilibrium, the factor prices are given by $\hat{r}_t = F_{k,t}$, $\hat{w}_t = F_{l,t}$. Conversely, for any tax-distorted equilibrium allocation under product market policies, we can construct prices and taxes so that this allocation is an equilibrium allocation without product market policies.

Proof. We show that the allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ is consistent with firm optimization, consumer optimization, consumer budget constraint, and government budget constraint in the decentralization with product market policies under appropriately defined prices and taxes. First, define product market policy as in (38). Define taxes as follows. For all $t \geq 1$:

$$1 - \hat{\tau}_{k,t} = \frac{\left(1 - \frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t}{F_{k,t} - \delta q_t}(1 - \tau_{k,t}) \tag{40}$$

and

$$1 - \hat{\tau}_{l,t} = \left(1 - \frac{1}{\varepsilon_t}\right)(1 - \tau_{l,t}). \tag{41}$$

We do not need to define profit income tax rate since it is equal to the tax rate on capital income.

Next, define prices as follows. For all $t \geq 1$:

$$\hat{p}_t = p_t, \tag{42}$$

$$\hat{r}_t = F_{k,t}, \tag{43}$$

$$\hat{w}_t = F_{l,t}. \tag{44}$$

We begin with the production side of the economy. The final good producer's problem is unchanged, so it still implies the same demand function:

$$y_{i,t} = y_t \xi_{i,t}^{-\varepsilon_t}. \tag{45}$$

Intermediate goods producers solve:

$$\hat{\pi}_{i,t} = \max_{\xi_{i,t},y_{i,t},k_{i,t},l_{i,t}} (1+\hat{\tau}_{s,t})\xi_{i,t}y_{i,t} - \hat{r}_t k_{i,t} - \hat{w}_t l_{i,t} - \hat{T}_t \tag{46}$$

$$s.t.$$

$$y_{i,t} = F_t(k_{i,t}, l_{i,t}) = \Big(\alpha_{k,t}(A_{k,t}k_{i,t})^\rho + \alpha_{l,t}(A_{l,t}l_{i,t})^\rho\Big)^{1/\rho}. \tag{47}$$

The intermediate good firm's problem can be solved in two steps. In the first step, for a given marginal cost of producing the good, $m_{i,t}$, the firm chooses price to maximize its profits:

$$\max_{\xi_{i,t}} (1+\hat{\tau}_{s,t})\xi_{i,t}y_{i,t} - m_{i,t}y_{i,t} - \hat{T}_t \quad s.t. \quad (45). \tag{48}$$

The solution to this problem implies a constant markup over marginal cost

$$(1+\hat{\tau}_{s,t})\xi_{i,t} = m_{i,t}\frac{\varepsilon_t}{\varepsilon_t - 1}. \tag{49}$$

In the symmetric equilibrium of the model, all varieties have the same production function and all intermediate goods firms make identical choices of inputs and prices. This implies $y_{i,t} = y_t$ and $\xi_{i,t} = 1$ for all $i \in [0,1]$. We therefore have the optimal marginal cost of producing one more intermediate good equals for all firms $m_{i,t} = M_t = \frac{\varepsilon_t - 1}{\varepsilon_t}(1+\hat{\tau}_{s,t}) = 1$ under the sales subsidy specified in (38).

In the second step, each firm chooses capital and labor to minimize the cost of producing intermediate good. The firms also make same input choices in the symmetric equilibrium, so we have $k_{i,t} = k_t$ and $l_{i,t} = l_t$. Marginal cost of producing one more unit using capital or labor at the optimum gives

$$\frac{\hat{r}_t}{F_{k,t}} = \frac{\hat{w}_t}{F_{l,t}} = m_t = 1, \tag{50}$$

which gives

$$\hat{r}_t = F_{k,t} \tag{51}$$

and

$$\hat{w}_t = F_{l,t}, \tag{52}$$

in line with the constructed factor prices in (43) and (44).

Using (48) and the constructed value of lump-sum tax (38), we can calculate

$$\hat{\pi}_t = (1+\hat{\tau}_{s,t})\xi_{i,t}y_{i,t} - m_{i,t}y_{i,t} - \hat{T}_t = y_t \frac{1}{\varepsilon_t} \frac{F_{k,t} - \delta q_t}{\left(1-\frac{1}{\varepsilon_t}\right) F_{k,t} - \delta q_t}. \tag{53}$$

Now, we turn to the consumer side. We know that the allocation being part of an equilibrium without product market policies implies that for all $t \geq 1$:

$$u_{c,t}q_t = \beta u_{c,t+1}\left[q_{t+1} + \left(\left(1-\frac{1}{\varepsilon_t}\right) F_{k,t+1} - \delta q_{t+1}\right)(1-\tau_{k,t+1})\right]. \tag{54}$$

This condition, together with the definition of capital taxes given by (40) imply

$$u_{c,t}q_t = \beta u_{c,t+1}\left[q_{t+1} + (F_{k,t+1} - \delta q_{t+1})(1-\hat{\tau}_{k,t+1})\right]. \tag{55}$$

Similarly, the allocation being part of an equilibrium without product market policies implies that for all $t \geq 1$:

$$-u_{l,t} = \left(1-\frac{1}{\varepsilon_t}\right) F_{l,t}(1-\tau_{l,t})u_{c,t}. \tag{56}$$

This condition, together with the definition of labor taxes given by (41) imply

$$-u_{l,t} = F_{l,t}(1-\hat{\tau}_{l,t})u_{c,t}. \tag{57}$$

(55) and (57) together imply that the original allocation satisfies consumer's intertemporal and intratemporal optimality condition when he faces newly constructed prices and taxes, (42)-(44) and (40)-(41). Next we show that consumer's budget constraint holds with equality under the original allocation and newly constructed prices and taxes. Since the original allocation is an equilibrium allocation without product market policies, it satisfies consumer's budget constraint in the no product market policies environment. That is,

$$\begin{aligned}&\sum_{t=1}^{\infty} p_t\left(c_t + q_t k_{t+1}\right) \\ &= \sum_{t=1}^{\infty} p_t\left(w_t l_t(1-\tau_{l,t}) + [q_t + (r_t - q_t\delta)(1-\tau_{k,t})]k_t + \pi_t(1-\tau_{\pi,t})\right) + p_1 b_1.\end{aligned} \tag{58}$$

Using (58), the definitions of intertemporal prices and rental and wage rates (42)-(44), and the definition of taxes (40)-(41), it follows that

$$\begin{aligned}&\sum_{t=1}^{\infty}\hat{p}_t\left(c_t+q_tk_{t+1}\right)\\&=\sum_{t=1}^{\infty}\hat{p}_t\left(\hat{w}_tl_t(1-\hat{\tau}_{l,t})+[q_t+(\hat{r}_t-q_t\delta)(1-\hat{\tau}_{k,t})]k_t+\hat{\pi}_t(1-\hat{\tau}_{\pi,t})\right)+\hat{p}_1b_1.\end{aligned}$$

So, consumer budget is satisfied.

Next, we need to show that the government budget constraint is satisfied under newly defined prices and taxes and the original allocation. In the original equilibrium, we have:

$$\sum_{t=1}^{\infty}p_tg_t+p_1b_1=\sum_{t=1}^{\infty}p_t\left(w_tl_t\tau_{l,t}+(r_t-q_t\delta)k_t\tau_{k,t}+\pi_t\tau_{\pi,t}\right). \tag{59}$$

First, notice that, in every period $t\geq 1$, the product market policy brings the government an additional fiscal burden equal to

$$\hat{T}_t-\hat{\tau}_{s,t}y_t=-\frac{1}{\varepsilon_t}\frac{F_{k,t}-\delta q_t}{\left(1-\frac{1}{\varepsilon_t}\right)F_{k,t}-\delta q_t}y_t. \tag{60}$$

Therefore, we need to show that

$$\sum_{t=1}^{\infty}\hat{p}_t\left(g_t+y_t\frac{1}{\varepsilon_t}\frac{F_{k,t}-\delta q_t}{\left(1-\frac{1}{\varepsilon_t}\right)F_{k,t}-\delta q_t}\right)+\hat{p}_1b_1=\sum_{t=1}^{\infty}p_t\left(\hat{w}_tl_t\hat{\tau}_{l,t}+(\hat{r}_t-q_t\delta)k_t\hat{\tau}_{k,t}+\hat{\pi}_t\hat{\tau}_{\pi,t}\right). \tag{61}$$

One can show that, by construction of new taxes and prices

$$(\hat{r}_t-q_t\delta)k_t\hat{\tau}_{k,t}-(r_t-q_t\delta)k_t\tau_{k,t}=\frac{1}{\varepsilon_t}F_{k,t}k_t \tag{62}$$

and

$$\hat{w}_tl_t\hat{\tau}_{l,t}-w_tl_t\tau_{l,t}=\frac{1}{\varepsilon_t}F_{l,t}l_t. \tag{63}$$

Furthermore,

$$\hat{\pi}_t\hat{\tau}_{\pi,t}-\pi_t\tau_{\pi,t}=\frac{1}{\varepsilon_t}\left(\frac{F_{k,t}-\delta q_t}{\left(1-\frac{1}{\varepsilon_t}\right)F_{k,t}-\delta q_t}-1\right)y_t. \tag{64}$$

Plugging (62)-(64) into (59), and using the fact that F is a constant returns to scale production function, we see immediately that (61) holds with equality. Finally, market clearing is implied by the fact that this allocation is an equilibrium allocation without product market policies. We have shown that the original allocation constitutes an equilibrium with proposed product market policies and under the newly constructed prices and taxes. □

Proof of Proposition 4.

Proof. Observing that the rental and wage rates equal the corresponding marginal products, the optimal capital and labor income tax rates that implement the Ramsey allocation in the market equilibrium with product market policies are defined by:

$$1-\tau_{k,t}^* = \frac{\frac{u_{c,t-1}^* q_{t-1}}{\beta u_{c,t}^*} - q_t}{F_{k,t}^* - \delta q_t}, \tag{65}$$

$$1-\tau_{l,t}^* = \frac{v_{l,t}^*}{u_{c,t}^* F_{l,t}^*}. \tag{66}$$

Combining (27) with the steady-state version of (65), and following the analogue of steps from the proof of Proposition 2 in Appendix A.2 delivers the capital tax formula (17). □

Steady-state optimal labor tax formula. Suppose $\tau_{k,t} = \tau_{\pi,t}$ for all $t \geq 1$. The long-run optimal tax rate on labor income is given by

$$\tau_l^* = 1 - \frac{1+\lambda^*\left(1+u_{cc}^* c^*/u_c^*\right)}{1+\lambda^*\left(1+u_{ll}^* l^*/u_l^*\right)}\left(1-\chi^* S_\pi (1-\tau_k^*)\left[1+\mathcal{E}_{1-\tau_k,l}^*/\tilde{S}_l^*\right]\right), \tag{67}$$

where $\mathcal{E}_{1-\tau_k,l}^* = \frac{d\ln(1-\tau_k)}{dl}|_{l=l^*}$ is the elasticity of the retention rate on capital income with respect to the equilibrium level of labor and $\tilde{S}_l^* = \frac{F_l^* l^*}{Y^*}$ is the inverse of labor's share in production.

Proof. Combining (31) and (32) with the steady-state version of (66), and rearranging gives the labor tax formula (67). □

A.5 Optimal Taxation with Exogenous Profit Taxes

Ramsey problem. Given (k_1, b_1), initial capital levy $\tau_{k,1} = \bar{\tau}_{k,1}$, the sequence of profit taxes $\{\bar{\tau}_{\pi,t}\}_{t=1}^{\infty}$, and a sequence of government spending $\{g_t\}_{t=1}^{\infty}$, government chooses allocation (c, k, l) to solve the following problem:

$$\max_{c,k,l} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t) \qquad \text{s.t.} \tag{68}$$

$$c_t + q_t k_{t+1} \leq F_t(k_t, l_t) + (1-\delta) q_t k_t, \quad \text{for all } t,$$

$$\sum_{t=1}^{\infty} \beta^{t-1} \left(u_{c,t} c_t + u_{l,t} l_t \right) = \sum_{t=1}^{\infty} \beta^{t-1} u_{c,t} (1 - \bar{\tau}_{\pi,t}) \pi_t + u_{c,1} (\bar{r}_1 k_1 + b_1),$$

where $\pi_t = \frac{1}{\varepsilon_t} F_t(k_t, l_t)$.

Proof of Proposition 5.

Proof. Letting $\beta^{t-1}\mu_t$ and λ be LaGrange multipliers on period t feasibility constraint and implementability constraint, the first-order conditions of Ramsey problem (68) for $t \geq 2$ are:

$$(k_t) : -\beta^{t-2} \mu_{t-1}^* q_{t-1} + \beta^{t-1} \mu_t^* \left(F_{k,t}^* + (1-\delta) q_t \right) - \lambda^* \beta^{t-1} u_{c,t}^* (1 - \bar{\tau}_{\pi,t}) \frac{\partial \pi_t^*}{\partial k_t} = 0, \tag{69}$$

$$(l_t) : \beta^{t-1} u_{l,t}^* + \lambda^* \beta^{t-1} \left[u_{ll,t}^* l_t + u_{l,t}^* \right] - \lambda^* \beta^{t-1} u_{c,t}^* (1 - \bar{\tau}_{\pi,t}) \frac{\partial \pi_t^*}{\partial l_t} + \beta^{t-1} \mu_t^* F_{l,t}^* = 0, \tag{70}$$

$$(c_t) : \beta^{t-1} u_{c,t}^* + \lambda^* \beta^{t-1} \left[u_{cc,t}^* c_t^* + u_{c,t}^* \right] - \lambda^* \beta^{t-1} u_{cc,t}^* (1 - \bar{\tau}_{\pi,t}) \pi_t^* - \beta^{t-1} \mu_t^* = 0. \tag{71}$$

Combining the steady-state versions of (69) and (65), and rearranging gives the capital tax formula (18). □

Steady-state optimal labor tax formula. Suppose there is a sequence of exogenous upper limits, $\{\bar{\tau}_{\pi,t}\}_{t=1}^{\infty}$, on profit taxes and suppose these limits converge to $\bar{\tau}_\pi$ over time. The long-run optimal tax rate on capital income is given by

$$\tau_l^* = 1 - \frac{1 + \lambda^* \left(1 + u_{cc}^* c^* / u_c^*\right)}{1 + \lambda^* \left(1 + u_{ll}^* l^* / u_l^*\right)} \left(1 - \chi^* S_\pi (1 - \bar{\tau}_\pi)\right). \tag{72}$$

Proof. Combining the steady-state versions of (70) and (71) with (66), and rearranging gives the labor tax formula (72). □

Discussion on capital subsidies in the short run. Recall that the Ramsey planner wants to minimize the net-present value of after-tax profits that appears on the right-hand-side of the implementability constraint in (68). A subsidy on period t capital income increases savings into period t, which increases consumption, and hence, decreases the equilibrium price of consumption in that period. Since period t profits accrue in period t prices, this decreases the net-present value of period t profits. This introduces a motive to subsidize period t capital income. Similarly, a tax on period t capital income increases consumption in $t-1$, and hence, reduces the value of $t-1$ after-tax profits, which introduces a motive to tax capital. The magnitude of these forces are proportional to the magnitude of after-tax profit income in each period. The subsidy motive dominates the tax motive early on, and we get capital subsidies to be optimal. The capital income tax turns positive after a while because the profit share grows large enough that the aforementioned price effects of taxing capital become too small relative to the indirect profit tax revenue benefit of capital taxation. We do not get negative taxes to be optimal even early on in the baseline case with uniform tax on capital and profit income because in that case the presence of the additional direct profit tax revenue effect of capital income taxation dominates the price effects from the start of the reform. Finally, the price effects do not appear in the steady-state formulas because the aforementioned subsidy and tax motives exactly offset each other in a steady state.

A.6 Proof of Proposition 6

Proof. Letting $\beta^{t-1}\mu_t$ and λ be LaGrange multipliers on period t feasibility constraint and implementability constraint, the first-order conditions of Ramsey problem (20) for $t \geq 2$ are:

$$(k_t) : -\beta^{t-2}\mu^*_{t-1}q_{t-1}+\beta^{t-1}\mu^*_t\left(\left(1-\frac{1}{\varepsilon_t}\right)F^*_{k,t}+(1-\delta)q_t\right) \\ -\lambda^*\beta^{t-1}u^*_{c,t}\left[(1-\tau^*_{\pi,t})\frac{\partial\pi^*_t}{\partial k_t}+\frac{\partial(1-\tau^*_{\pi,t})}{\partial k_t}\pi^*_t\right]=0, \tag{73}$$

$$(l_t) : \beta^{t-1}u^*_{l,t}+\lambda^*\beta^{t-1}\left[u^*_{ll,t}l_t+u^*_{l,t}\right] \\ -\lambda^*\beta^{t-1}u^*_{c,t}\left[(1-\tau^*_{\pi,t})\frac{\partial\pi^*_t}{\partial l_t}+\frac{\partial(1-\tau^*_{\pi,t})}{\partial l_t}\pi^*_t\right]+\beta^{t-1}\mu^*_t\left(1-\frac{1}{\varepsilon_t}\right)F^*_{l,t}=0, \tag{74}$$

$$(c_t) : \beta^{t-1} u^*_{c,t} + \lambda^* \beta^{t-1} \left[u^*_{cc,t} c^*_t + u^*_{c,t} \right] - \lambda^* \beta^{t-1} u^*_{cc,t} (1 - \tau^*_{\pi,t}) \pi^*_t$$
$$- \lambda^* \beta^{t-1} \left[u^*_{c,t} \frac{\partial (1 - \tau^*_{\pi,t})}{\partial c_t} \pi^*_t + \beta u^*_{c,t+1} \frac{\partial (1 - \tau^*_{\pi,t+1})}{\partial c_t} \pi^*_{t+1} \right] - \beta^{t-1} \mu^*_t = 0. \quad (75)$$

Combining the steady-state versions of (73) and (8), and and following the analogue of steps from the proof of Proposition 2 in Appendix A.2 delivers the capital tax formula (21). □

Steady-state optimal labor tax formula. Suppose $\tau_{k,t} = \tau_{\pi,t}$ for all $t \geq 1$. The long-run optimal tax rate on labor income is given by

$$\tau^*_l = 1 - \frac{1 + \lambda^* \left(1 + u^*_{cc} c^* / u^*_c\right)}{1 + \lambda^* \left(1 + u^*_{ll} l^* / u^*_l\right)} \left(1 - \chi^* \frac{S_\pi}{1 - S_\pi} (1 - \tau^*_k) \left[1 + \mathcal{E}^*_{1-\tau_k, l} / \tilde{S}^*_l \right] \right), \quad (76)$$

where $\mathcal{E}^*_{1-\tau_k,l} = \frac{d\ln(1-\tau_k)}{dl}|_{l=l^*}$ is the elasticity of the retention rate on capital income with respect to the equilibrium level of labor and $\tilde{S}^*_l = \frac{F^*_l l^*}{Y^*}$ is the inverse of labor's share in production.

Proof. Combining the steady-state versions of (78) and (79), and (9), and rearranging gives the labor tax formula. □

A.7 Non-Separable Preferences

We first derive the first-order optimality conditions of Ramsey Problem (14) under non-separable preferences. Let $\beta^{t-1} \mu_t$ and λ be LaGrange multipliers on period t feasibility constraint and implementability constraint. The first-order optimality condition for k_t remains the same as in (24). Assuming that the allocation and the multiplier on the resource constraint converges, at the steady state (24) becomes

$$(k) : -\mu^* q + \mu^* \left(F^*_k + (1 - \delta) q \right) - \lambda^* u^*_c \left[(1 - \tau^*_\pi) \frac{\partial \pi^*}{\partial k} + \frac{\partial (1 - \tau^*_\pi)}{\partial k} \pi^* \right] = 0. \quad (77)$$

Combining (77) with the steady-state version of (8), and rearranging gives the optimal steady-state capital tax formula (16) in Proposition 2.

The first-order conditions with respect to labor and consumption are as follows. For

$t \geq 2:$

$$(l_t): \beta^{t-1} u^*_{l,t} + \lambda^* \beta^{t-1} \left[u^*_{lc,t} c_t + u^*_{ll,t} l_t + u^*_{l,t} - u^*_{lc,t}(1-\tau_{\pi,t})\pi_t \right]$$
$$-\lambda^* \beta^{t-1} u^*_{c,t} \left[(1-\tau^*_{\pi,t}) \frac{\partial \pi^*_t}{\partial l_t} + \left\{ \frac{\partial (1-\tau^*_{\pi,t})}{\partial F_{k,t}} \frac{\partial F_{k,t}}{\partial l_t} + \frac{\partial (1-\tau^*_{\pi,t})}{\partial u_{c,t}} \frac{\partial u_{c,t}}{\partial l_t} \right\} \pi^*_t \right]$$
$$- \lambda^* \beta^t u^*_{c,t+1} \frac{\partial (1-\tau^*_{\pi,t+1})}{\partial l_t} \pi^*_{t+1} + \beta^{t-1} \mu^*_t F^*_{l,t} = 0, \tag{78}$$

$$(c_t): \beta^{t-1} u^*_{c,t} + \lambda^* \beta^{t-1} \left[u^*_{cc,t} c^*_t + u^*_{c,t} + u^*_{cl,t} l^*_t \right] - \lambda^* \beta^{t-1} u^*_{cc,t} (1-\tau^*_{\pi,t}) \pi^*_t$$
$$- \lambda^* \beta^{t-1} \left[u^*_{c,t} \frac{\partial (1-\tau^*_{\pi,t})}{\partial c_t} \pi^*_t + \beta u^*_{c,t+1} \frac{\partial (1-\tau^*_{\pi,t+1})}{\partial c_t} \pi^*_{t+1} \right] - \beta^{t-1} \mu^*_t = 0. \tag{79}$$

At the steady state (78) and (79) become

$$(l): u^*_l + \lambda^* \left[u^*_{lc} c^* + u^*_{ll} l^* + u^*_l \right] - \lambda^* u^*_c \left[(1-\tau^*_\pi) \frac{\partial \pi^*}{\partial l} + \frac{\partial (1-\tau^*_\pi)}{\partial l} \pi^* \right] + \mu^* F^*_l = 0 \tag{80}$$

and

$$(c): u^*_c + \lambda^* \left[u^*_{cc} c^* + u^*_c + u^*_{cl} l^* \right] - \mu^* = 0, \tag{81}$$

where $\frac{\partial(1-\tau^*_\pi)}{\partial l} = \frac{\partial(1-\tau^*_\pi)}{\partial F_k} \frac{\partial F_k}{\partial l}$, which is identical to $\frac{\partial(1-\tau^*_\pi)}{\partial l}$ in the case with separable utility. However, as we can see already by glancing (80) and (81), the cross derivative utility terms are new relative to the non-separable case. Combining (80) and (81) with the steady-state version of (9), and rearranging gives the labor tax formula for non-separable case:

$$\tau^*_l = 1 - \frac{1 + \lambda^* \left(1 + u^*_{cc} c^* / u^*_c + u^*_{cl} l^* / u^*_c\right)}{1 + \lambda^* \left(1 + u^*_{ll} l^* / u^*_l + u^*_{lc} c^* / u^*_l\right)} \left(1 - \chi^* S_\pi (1-\tau^*_k) \left[1 + \mathcal{E}^*_{1-\tau_k, l} / \tilde{S}^*_l \right]\right) \frac{1}{1 - S_\pi}. \tag{82}$$

A comparison of (82) with (30) reveals that although the optimal steady-state labor tax formulas are modified by non-separability, the modification appears in the standard Ramsey term of labor taxes and not in terms related to un-taxed profits.

B Data Construction

B.1 Construction of Labor and Capital Share Series

In this section, our construct labor, capital and profit share series for the U.S. Non-Farm Business Sector. Following Barkai and Benzell (2018), Barkai (2020), Karabarbounis and Neiman (2018), and De Loecker, Eeckhout, and Unger (2020), profits are measured as a residual after deducting labor costs and capital costs from gross value added. The labor, capital, and profit shares are defined, respectively, as the ratio of capital costs, labor costs, and profits to gross value added as follows:

$$1 = S_{L,t} + S_{K,t} + S_{\pi,t}$$

where $S_{L,t} = \frac{w_t L_t}{Y_t}$ is the labor share, $S_{K,t} = \frac{R_t K_t}{Y_t}$ is the capital share and $S_{\pi,t} = \frac{\Pi_t}{Y_t}$ is the profit share.

The data for the U.S. Non-Farm Business Sector economy comes from the Bureau of Economic Analysis (BEA) NIPA and Fixed Asset Tables, Bureau of Labor Statistics (BLS), Federal Reserve Board (FED) Financial Accounts and the St. Louis FED Database (FRED).

Labor Share: The data on *Labor Share in the U.S. Non-Farm Business Sector* - S_t^L - is reported annually by the BLS.

Capital Share: The capital share is calculated as the imputed capital rental payments $R_t K_t$ as a share of gross value added Y_t. Gross value added and capital stock (structures, equipment, intellectual property, inventories) data are taken from BEA and FED Flow of Funds. Rental rate R_t is constructed by using data on real rate r_t (see below), capital price q_t, depreciation rate δ_t, the tax rate on capital τ_t^k (effective average tax rate) and expected inflation $E(\pi)$ (5 year moving average of realized inflation on consumption expenditures). Following Barkai (2020), the real rate is calculated as

$$r_t = [(1 - \tau_t^k)\frac{D}{D+E} r_t^D + \frac{E}{D+E} r_t^E] - E(\pi_t),$$

where D is the stock of debt, E is the stock of equity, r^D is the nominal return on debt, r^E is the nominal return on equity, and the real rate is the weighted average of nominal returns on debt and equity corrected with inflation expectations. The data on (i) stock of debt and equity are from Fed Flow of Funds, (ii) r^D is the Moody's AAA bond yield, (iii) r^E is the yield on U.S. treasury bonds plus a 5% equity risk premium as in Barkai (2020).

Based on this, one can drive the formula for rental rate within our model (or a standard one sector growth model) as follows:

$$R_t = \frac{1}{(1-\tau_t^k)}.[q_{t-1}.(1+r_t) - q_t.(1-(1-\tau_t^k)\delta_t)]$$

which is consistent with the derivation of Karabarbounis and Neiman (2018). Accordingly, we calculate capital share $S_{K,t}$ as $\frac{R_t K_t}{Y_t}$.

Profit Share: We derive the profit share $S_{\pi,t}$ for the US Non-Farm Business Sector as $= 1 - S_{K,t} - S_{L,t}$. Figure 11 plots (i) our own construction of the profit share series for the US Non-Farm Business Sector, (ii) the time series evolution of profit share in our model, and, for comparison, (iii) the profit share series constructed by Karabarbounis and Neiman (2018).

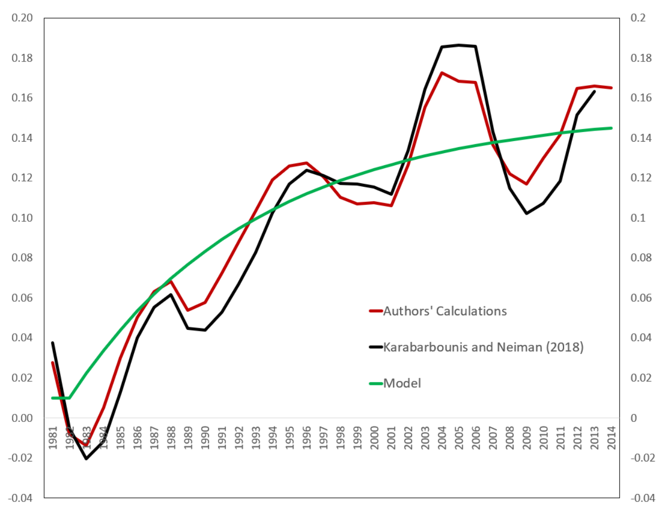


Figure 11: Profit Share

This figure reports the empirical profit share series that is constructed by the authors, by Karabarbounis and Neiman (2018), and the profit share series that the benchmark calibration delivers.

B.2 Construction of Capital-Output, Profit-Capital, Gross Profitability and $APK - R$ Series

Capital-output ratio. Output series for non-farm business sector are from BLS (labor productivity and related measures tables). Capital series (composed of equipment, non-housing structures, intellectual property and inventory) are constructed using BEA Fixed Asset Tables.

Profit-capital ratio. Profit-capital ratio is the ratio of profit income to capital stock. Profit series are calculated as described in Appendix C.1.

Gross profitability. Gross Profitability is defined as 1-Labor Share divided by Capital-Output Ratio. Labor Share series for non-farm business sector are from BLS (labor productivity and related measures tables).

APK-R. The data on APK series are generated for the US corporate sector based on BEA NIPA and Fixed Asset Tables and the methodology closely follows Caballero, Farhi, and Gourinchas (2017). Our baseline measure of R is the nominal rate on 10-year U.S. Treasuries minus 5-year moving average of realized inflation that proxies expected inflation, as in Karabarbounis and Neiman (2018).

The difference between these two variables is given by the following expression in the model:

$$APK_t - R_t = \frac{q_{t-1} - q_t}{q_{t-1}} + \frac{Y_t \frac{1}{\varepsilon_t}(1 - \tau_{k,t})}{q_{t-1} K_t}. \tag{83}$$

We can see from (83) that, among other factors, higher profit share leads to an increase in $APK - R$. In fact, our simulations show that the share of profits in the economy is the dominant factor in determining $APK - R$.

C Alternative Calibration of Exogenous Profit Taxes

Recall that, in the benchmark calibration of Section 4.2, the profit tax rate τ_π decreases from 40% to 30% following the observed capital income tax series. As a sensitivity analysis, we also consider an alternative calibration in which the time-series for τ_π is equalized to the observed tax rate on distributions (see McGrattan and Prescott (2010)), decreasing from a level of 40% to 15% over the period of interest. Figure 12 plots the τ_π time-series used in our benchmark and alternative parameterizations. We recalibrate the model under this time series for profit taxes using the same methodology as in Section 4.2.

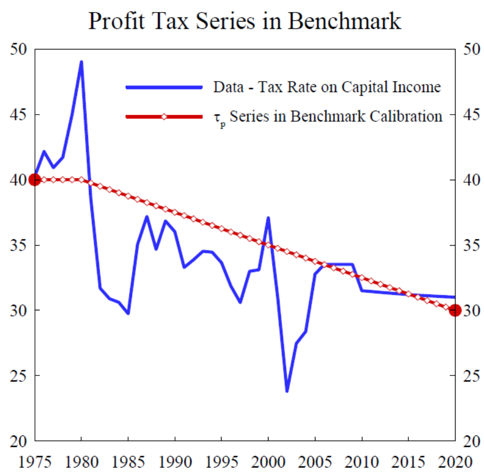


Figure 12: (a)

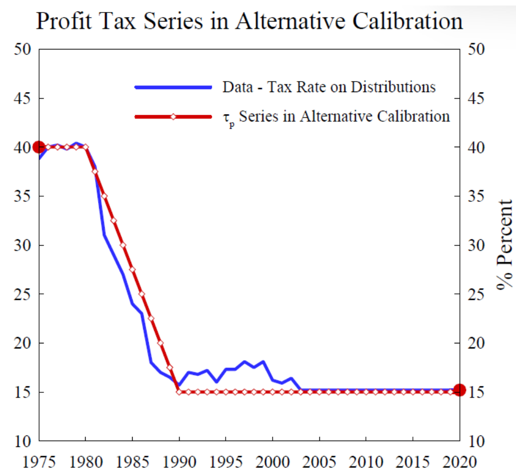


Figure 12: (b)

The figure depicts profit tax series for the benchmark calibration (a) and for an alternative calibration in which profit tax rate is assumed to equal the observed tax rate on distributions as in McGrattan and Prescott (2010).

Figure 13 illustrates the time-path of optimal capital income taxes for both calibrations. Under alternative calibration, we find that optimal capital income taxes are again positive and somewhat higher than the ones implied by the benchmark calibration. We also observe a similar pattern for the optimal labor income taxes under the alternative calibration. To sum up, our quantitative findings verify that, under alternative assumptions on the evolution of actual τ_π series, the optimal capital income taxes are still significantly positive in case of exogenous profit taxes.

D Multi-Sector Product Market Model

In an influential paper, De Loecker, Eeckhout, and Unger (2020) find that the rise in aggregate profits (and markups) is mainly driven by the right tail of the profit distribution. In particular, while the profit rates in the upper percentiles of the distribution have increased sharply, the profit rates of the median firm has not changed since the early 1980's. They also show that, in addition to fattening of the right tail of the unweighted markup distribution, there has been a reallocation of market share from low to high markup firms.

By construction, our single-sector monopolistic competition model introduced in Section 2

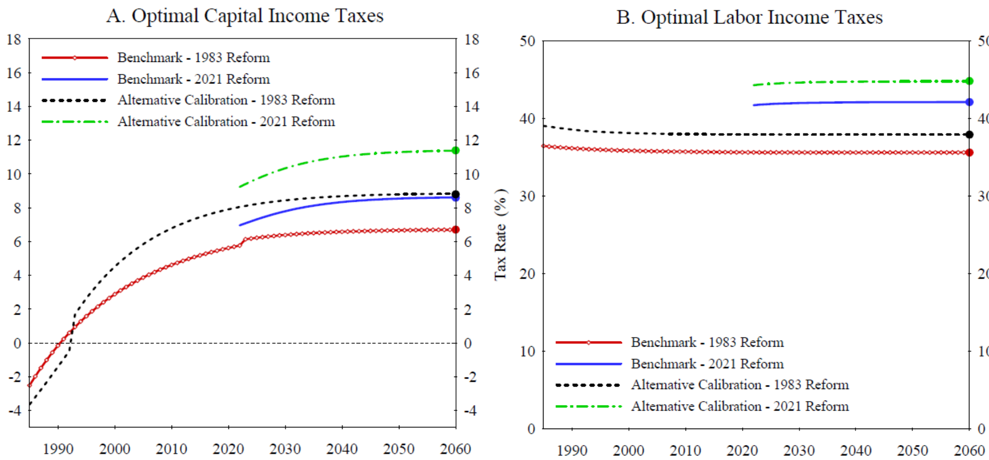


Figure 13: Optimal Taxes under Exogenous Profit Taxes: Alternative Calibration

This figure depicts the time series of the optimal capital income tax rates (a) and the optimal labor income tax rates (b) for the case in which profits are taxed separately from capital income at an exogenous rate and product market policies are available for the benchmark calibration and for an alternative calibration in which profit tax rate is assumed to equal the observed tax rate on distributions as in McGrattan and Prescott (2010).

cannot capture the heterogeneity in profit rates and its evolution over time. In this appendix, we consider a multi-sector extension that is able to mimic the broad patterns described by De Loecker, Eeckhout, and Unger (2020). In what follows, we first lay out the multi-sector extension and derive the Ramsey problem for this model. Then, we calibrate the model to be in line with (i) the calibration targets of the single-sector model in terms of the evolution of aggregate income shares and (ii) some moments of the empirical distribution of profit rates in the early 1980's and late 2010's. Then, we display quantitative optimal tax results for this economy. For brevity, we only conduct the analysis for the baseline case in which the government only has access to capital and labor income taxes (no product market policies) and the tax rate on capital and profits have to be the same.

D.1 Multi-Sector Product Market

Consider a model in which there are $j = 1, 2, ..., N$ sectors and in each sector there is a unit measure of monopolistically competitive intermediate goods firms.

Final Good Producers. Firms that produce the final good are perfectly competitive and operate a constant elasticity of substitution (CES) production function that combines intermediate goods $y_{j,i,t}$. Taking prices of intermediate goods, $\xi_{j,i,t}$, as given, the problem of the representative final good producer is:

$$\max_{y_{i,t}} y_t - \sum_{j=1}^{N} \int_0^1 \xi_{j,i,t} y_{j,i,t} di \qquad \text{s.t.} \qquad y_t = \left(\sum_{j=1}^{N} y_{j,t}^{\frac{\gamma_t - 1}{\gamma_t}} \right)^{\frac{\gamma_t}{\gamma_t - 1}} \text{ and } y_{j,t} = \left(\int_0^1 y_{j,i,t}^{\frac{\varepsilon_{j,t}-1}{\varepsilon_{j,t}}} di \right)^{\frac{\varepsilon_{j,t}}{\varepsilon_{j,t}-1}}.$$

Intermediate Good Producers. Each intermediate good producer is a monopolistic competitor. Producer of intermediate good $y_{j,i,t}$ uses a CES technology, $F_{j,t}$, to combine capital and labor to produce the intermediate good. This firm solves:

$$\pi_{j,i,t} = \max_{\xi_{j,i,t}, y_{j,i,t}, k_{j,i,t}, l_{j,i,t}} \xi_{j,i,t} y_{j,i,t} - r_t k_{j,i,t} - w_t l_{j,i,t} \qquad \text{s.t.} \tag{84}$$

$$y_{j,i,t} = F_{j,t}(k_{j,i,t}, l_{j,i,t}) \text{ and } (85),$$

where, as in the single-sector case, r_t and w_t represent the real rental rates of capital and labor, respectively. The rest of the model is identical to the one introduced in Section 2.

D.2 Equilibrium Characterization

The definition of equilibrium is similar to the definition of market equilibrium given in Section 2, and hence, is omitted. The first-order optimality condition of the final good producer's problem with respect to intermediate good $y_{j,i,t}$ gives its demand as a function of price:

$$y_{j,i,t} = \left(\sum_{j=1}^{N} y_{j,t}^{\frac{\gamma_t - 1}{\gamma_t}} \right)^{\frac{\varepsilon_{j,t}}{\gamma_t - 1}} y_{j,t}^{-\frac{\varepsilon_{j,t}}{\gamma_t}+1} \xi_{j,i,t}^{-\varepsilon_{j,t}}. \tag{85}$$

We are going to assume that

$$y_{j,i,t} = z_{j,t} F_t(k_{j,i,t}, l_{j,i,t}), \tag{86}$$

where F_t is a constant returns to scale (CRS) technology that is common to all sectors and $z_{j,t}$ is sector-specific productivity that evolves over time deterministically.

The problem of the intermediate good producer can be solved in two steps. In the first step, for a given marginal cost of producing the intermediate good, $m_{j,i,t}$, the firm chooses its price to maximize profits:

$$\max_{\xi_{j,i,t}} \; \xi_{j,i,t} y_{j,i,t} - m_{j,i,t} y_{j,i,t} \quad s.t. \quad (85). \tag{87}$$

The solution to this problem implies a constant markup over marginal cost that is the same for all the firms in a sector:

$$\xi_{j,i,t} = m_{j,i,t} \frac{\varepsilon_{j,t}}{\varepsilon_{j,t} - 1}. \tag{88}$$

We focus on the symmetric equilibrium of the model in which all intermediate goods firms in a given sector make identical choices of inputs and prices. This implies $y_{j,i,t} = y_{j,t}$ and $\xi_{j,i,t} = \xi_{j,t}$ for all $i \in [0,1]$. We, therefore, have that the optimal marginal cost of producing one more intermediate good equals

$$m_{j,i,t} = m_{j,t} = \xi_{j,t} \frac{\varepsilon_{j,t} - 1}{\varepsilon_{j,t}}.$$

In the second step, each firm chooses capital and labor to minimize the cost of production. The firms also make same input choices in the symmetric equilibrium, so we have $k_{j,i,t} = k_{j,t}$ and $l_{j,i,t} = l_{j,t}$. The marginal cost of producing one more unit of the intermediate good using capital or labor at the optimum equals $\frac{r_t}{z_{j,t} F_k(k_{j,t}, l_{j,t})} = \frac{w_t}{z_{j,t} F_l(k_{j,t}, l_{j,t})} = \xi_{j,t} \frac{\varepsilon_{j,t} - 1}{\varepsilon_{j,t}}$, where $F_k(k_{j,t}, l_{j,t})$ is short-hand notation for $\frac{\partial F_t(k_{j,t}, l_{j,t})}{\partial k_{j,t}}$ and $F_l(k_{j,t}, l_{j,t})$ is defined analogously. Therefore, the rental and wages rate are given by

$$r_t = \left(1 - \frac{1}{\varepsilon_{j,t}}\right) z_{j,t} F_k(k_{j,t}, l_{j,t}) \xi_{j,t} \qquad \text{and} \qquad w_t = \left(1 - \frac{1}{\varepsilon_{j,t}}\right) z_{j,t} F_l(k_{j,t}, l_{j,t}) \xi_{j,t}. \tag{89}$$

This implies that

$$\frac{r_t}{w_t} = \frac{F_k(k_{j,t}, l_{j,t})}{F_l(k_{j,t}, l_{j,t})} = \frac{F_k(k_{j,t}/l_{j,t}, 1)}{F_l(k_{j,t}/l_{j,t}, 1)}, \tag{90}$$

where the second equality follows from F being CRS. This implies that $k_{j,t}/l_{j,t} := \kappa_t$ is independent of j. Using this in (89) for any two sectors j and m, we get

$$\eta_{j,t} \xi_{j,t} = \eta_{m,t} \xi_{m,t}, \tag{91}$$

where $\eta_{j,t} = \left(1 - \frac{1}{\varepsilon_{j,t}}\right) z_{j,t}$ is a summary statistic that is increasing in a sector's productivity and within-sector competitiveness (defined as the inverse of the markup in that sector). The equilibrium condition (91) implies that the price of the goods produced by firms in sector j relative to the price of the sector m good is decreasing in relative productivity of sector j and relative competitiveness of sector j.

The final good producer's optimality condition (85) can be used to show that

$$\frac{\xi_{j,t} y_{j,t}}{y_t} = \left(\frac{y_{j,t}}{y_t}\right)^{1-\frac{1}{\gamma_t}}. \tag{92}$$

One can use (92) to show that $\xi_{j,t} = (\frac{y_{j,t}}{y_t})^{-\frac{1}{\gamma_t}}$. Combining this with (91), we get

$$\frac{y_{j,t}}{y_{m,t}} = \left(\frac{\eta_{j,t}}{\eta_{m,t}}\right)^{\gamma_t}, \tag{93}$$

which, using the fact that, $y_{j,t} = z_{j,t} l_{j,t} F(\kappa_t, 1)$, implies

$$\frac{l_{j,t}}{l_{m,t}} = \left(\frac{\eta_{j,t}}{\eta_{m,t}}\right)^{\gamma_t} \frac{z_{m,t}}{z_{j,t}} = \frac{k_{j,t}}{k_{m,t}}. \tag{94}$$

Equilibrium conditions (93) and (94) establish that a sector's size is related to their $\eta_{j,t}$. These equations reveal that the relative size of the sectors depend on their relative competitiveness and productivity: the sectors that are more competitive and productive are larger.

Aggregate production function. Notice that at the symmetric equilibrium

$$\begin{aligned} y_t = \left(\sum_{j=1}^{N} y_{j,t}^{\frac{\gamma_t-1}{\gamma_t}}\right)^{\frac{\gamma_t}{\gamma_t-1}} &= \left(\sum_{j=1}^{N} (z_{j,t} F(k_{j,t}, l_{j,t}))^{\frac{\gamma_t-1}{\gamma_t}}\right)^{\frac{\gamma_t}{\gamma_t-1}} \\ &= \left(\sum_{j=1}^{N} (z_{j,t} l_{j,t})^{\frac{\gamma_t-1}{\gamma_t}} F(\kappa_t, 1)^{\frac{\gamma_t-1}{\gamma_t}}\right)^{\frac{\gamma_t}{\gamma_t-1}} \\ &= \left(\sum_{j=1}^{N} \left(\frac{z_{j,t} l_{j,t}}{l_t}\right)^{\frac{\gamma_t-1}{\gamma_t}} F(k_t, l_t)^{\frac{\gamma_t-1}{\gamma_t}}\right)^{\frac{\gamma_t}{\gamma_t-1}}, \end{aligned}$$

where $l_t = \sum_{m=1}^{N} l_{m,t}$ and $k_t = \sum_{m=1}^{N} k_{m,t}$ are aggregate labor and capital. One can then use (94) to arrive at the aggregate production function

$$y_t = z_t F(k_t, l_t), \text{ where } z_t = \left(\sum_{j=1}^{N} \left(\sum_{m=1}^{N} \left(\frac{\eta_{m,t}}{\eta_{j,t}}\right)^{\gamma_t} z_{m,t}^{-1}\right)^{\frac{1-\gamma_t}{\gamma_t}}\right)^{\frac{\gamma_t}{\gamma_t-1}}. \tag{95}$$

Factor prices. One can alternatively express aggregate output as

$$
\begin{aligned}
y_t &= \left(\sum_{m=1}^{N} (z_{m,t} F(k_{m,t}, l_{m,t}))^{\frac{\gamma_t - 1}{\gamma_t}} \right)^{\frac{\gamma_t}{\gamma_t - 1}} \\
&= \left(\sum_{m=1}^{N} \left(\frac{z_{m,t} l_{m,t}}{z_{j,t} l_{j,t}} \right)^{\frac{\gamma_t - 1}{\gamma_t}} (z_{j,t} F(k_{j,t}, l_{j,t}))^{\frac{\gamma_t - 1}{\gamma_t}} \right)^{\frac{\gamma_t}{\gamma_t - 1}} \\
&= \left(\sum_{m=1}^{N} \left(\frac{\eta_{m,t}}{\eta_{j,t}} \right)^{\gamma_t - 1} \right)^{\frac{\gamma_t}{\gamma_t - 1}} y_{j,t}. \qquad (96)
\end{aligned}
$$

Using this and that $\xi_{j,t} = (\frac{y_{j,t}}{y_t})^{-\frac{1}{\gamma_t}}$, we get that

$$
\xi_{j,t} = \left(\sum_{m=1}^{N} \left(\frac{\eta_{m,t}}{\eta_{j,t}} \right)^{\gamma_t - 1} \right)^{\frac{1}{\gamma_t - 1}}. \qquad (97)
$$

Using (97) in (89) delivers factor prices for capital and labor:

$$
r_t = \left(\sum_{j=1}^{N} \eta_{j,t}^{\gamma_t - 1} \right)^{\frac{1}{\gamma_t - 1}} F_k(k_t, l_t) \qquad \text{and} \qquad w_t = \left(\sum_{j=1}^{N} \eta_{j,t}^{\gamma_t - 1} \right)^{\frac{1}{\gamma_t - 1}} F_l(k_t, l_t). \qquad (98)
$$

Profit share. Recall that

$$
\pi_{j,t} = \frac{1}{\varepsilon_{j,t}} \xi_{j,t} y_{j,t} = \frac{1}{\varepsilon_{j,t}} \left(\sum_{m=1}^{N} \left(\frac{\eta_{m,t}}{\eta_{j,t}} \right)^{\gamma_t - 1} \right)^{-1} y_t, \qquad (99)
$$

where we plugged in $y_{j,t}$ and $\xi_{j,t}$ using (96) and (97). This implies

$$
\pi_t = \sum_{j=1}^{N} \varepsilon_{j,t}^{-1} \left(\sum_{m=1}^{N} \left(\frac{\eta_{m,t}}{\eta_{j,t}} \right)^{\gamma_t - 1} \right)^{-1} z_t F_t(k_t, l_t). \qquad (100)
$$

Since the rest of the economy (the consumer side and the government) are identical to the model laid out in Section 2, the characterization of that part is identical to that given in Section 2. As a result, we face the same economy as in Section 2 with an aggregate production function, factor prices and aggregate profits given by (95), (98) and (100).

D.3 Ramsey Problem

The following proposition, which is analogous to Proposition 1, establishes that the set of equilibrium allocations of this economy can be fully characterized by the set of constraints given in Proposition 1 but modified to take into account the new aggregate production function, factor prices and aggregate profits given by (95), (98) and (100). The proof of Proposition 8 is omitted as it closely follows that of Proposition 1.

Proposition 8. *If an allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ is part of a tax-distorted market equilibrium, then it satisfies the resource feasibility constraint (101), and the constraints (102) and (103) below. Conversely, suppose an allocation $\{c_t, k_{t+1}, l_t\}_{t=1}^{\infty}$ satisfies (101), (102) and (103). Then, we can construct prices and taxes such that this allocation together with constructed prices and taxes constitute an equilibrium allocation.*

$$c_t + q_t k_{t+1} + g_t = z_t F_t(k_t, l_t) + (1-\delta) q_t k_t, \tag{101}$$

$$\sum_{t=1}^{\infty} \beta^{t-1} \left(u_{c,t} c_t + u_{l,t} l_t \right) = \sum_{t=1}^{\infty} \beta^{t-1} u_{c,t} \pi_t (1 - \tau_{\pi,t}) + u_{c,1}(\bar{r}_1 k_1 + b_1), \tag{102}$$

$$\tau_{\pi,t} = 1 - \frac{\frac{u_{c,t-1} q_{t-1}}{\beta u_{c,t}} - q_t}{\left(\sum_{j=1}^{N} \eta_{j,t}^{\gamma_t - 1} \right)^{\frac{1}{\gamma_t - 1}} F_k(k_t, l_t) - \delta q_t}, \quad \forall t \geq 2, \tag{103}$$

where z_t and π_t are given by (95) and (100), respectively.

Ramsey problem. Given (k_1, b_1), initial policies $\tau_{\pi,1} = \tau_{k,1} = \bar{\tau}_{k,1}$, and a sequence of government spending, $\{g_t\}_{t=1}^{\infty}$, the government chooses allocation (c, k, l) to solve the problem:

$$\max_{c,k,l} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t) \qquad \text{s.t.} \tag{104}$$
$$(101), (102), \text{ and } (103).$$

The first-order optimality conditions with respect to capital, labor and consumption are identical to those in Appendix A.2, with profit π_t, production function $z_t F(k_t, l_t)$, and retention rate on profit income, $1 - \tau_{\pi,t}$, defined as above. The optimal capital and labor tax are then calculated by comparing the first-order optimal conditions of the Ramsey problem with the corresponding equilibrium optimality conditions for capital and labor, (105) and

(106) below, which are obtained by combining consumer's first-order optimality conditions with rental rates, which are now given by (98):

$$u_{c,t-1}q_{t-1} = \beta u_{c,t}\left[q_t + \left(\left(\sum_{j=1}^{N}\eta_{j,t}^{\gamma_t-1}\right)^{\frac{1}{\gamma_t-1}}F_{k,t} - \delta q_t\right)(1-\tau_{k,t})\right], \tag{105}$$

$$\left(\sum_{j=1}^{N}\eta_{j,t}^{\gamma_t-1}\right)^{\frac{1}{\gamma_t-1}}F_{l,t}(1-\tau_{l,t})u_{c,t} = -u_{l,t}. \tag{106}$$

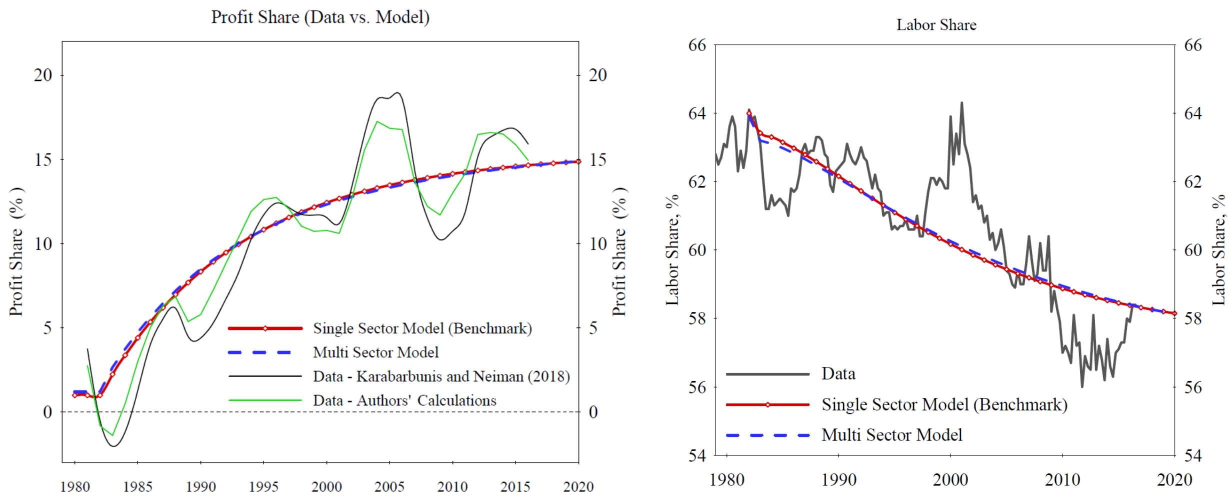
D.4 Calibration

All the parameters in Table 1 but ε are also present in the extended model. We set all these parameters to their values in Table 1. Without loss of generality, we set $\gamma_t = 2$ for all t. For each period, we need to specify a distribution of sectors over $(z_{j,t}, \varepsilon_{j,t})$ pairs. We do this so that i) aggregate z_t given by (95) is normalized to 1 every period t as in the single-sector model, ii) the time-series of the aggregate profit share given by (100) tracks the one from the single-sector model (red curve in Figure 3A), iii) the profit rate for the median firm stays roughly at the same level (2%) over time, and (iv) the profit rate for the 95th percentile firm increases from 18% in the early 1980's to 36% in the late 2010's. Empirical targets for iii) and iv) are taken from De Loecker, Eeckhout, and Unger (2020). Intentionally, we remain agnostic about what a sector corresponds to in the data.

To match the model and the data regarding empirical facts iii) and iv), we need a mapping between firm level profit rate in the model and firm level profit rate as defined by De Loecker, Eeckhout, and Unger (2020) who define profit rate of a firm as firm profits (which equal sales minus all production costs) divided by sales. In our model, this corresponds exactly to

$$\frac{\pi_{j,t}}{\xi_{j,t}y_{j,t}} = \varepsilon_{j,t}^{-1}.$$

We need to also specify the time series of $\alpha_{k,t}$. Instead of recalibrating this series for the multisector model, we feed in the same series that is calibrated to match the evolution of labor share in the single-sector model, which is plotted in Figure 3B. Figure 14A and Figure 14B plot the time series of profit and labor shares, implied by the the single-sector model (red line) and multi-sector model (dashed blue line). The figure reveals that the two models essentially generate the same income share dynamics, which is important for the



(a) Profit Share

(b) Labor Share

Figure 14: Evolution of income shares: single-sector vs. multi-sector models

Panel (a) plots the time series of the profit share observed in the data as measured by the authors, according to Karabarbounis and Neiman (2018), implied by the single-sector model and implied by the multi-sector model. Panel (b) plots the same for labor share.

comparability of optimal taxes across the two economies.

Figure 15 reports optimal taxes for the single-sector and multi-sector models. For brevity, we only report results for the 1983 reform and for the baseline case in which there is a uniform tax rate on capital and profit income and product market policies are not available. The figure shows that optimal capital and labor tax rates are very similar across the single-sector and multi-sector models.

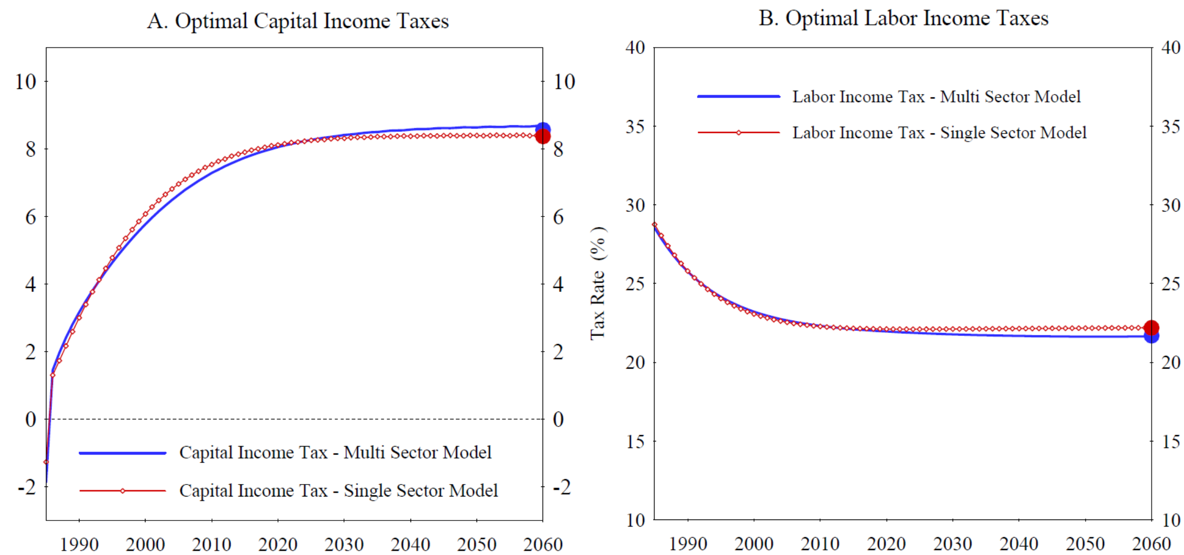


Figure 15: Optimal tax rates: single-sector vs. multi-sector models

This figure reports optimal capital and labor income tax rates for the 1983 reform for the single-sector and the multi-sector models.