Online Appendix for "Optimal Subsidization of Business Start-ups"

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A Proof of Proposition 1

Showing that the allocation described in Proposition 1 is in the constraint set of the planner's problem is sufficient since under this allocation total output is equal to the full information total output level.

Choose $\delta_1^*(w,i)$ for $(w,i) \neq (p,1)$ such that:

$$\sum_{(w,i)\neq(p,1)} \zeta_w \eta_i \delta_1^*(w,i) = -(\bar{k}-p)\zeta_p \eta_1, \tag{1}$$

$$\delta_1^*(r,1) \geq \bar{k} - r, \tag{2}$$

$$0 \geq \delta_1^*(w,0) \geq -w, \tag{3}$$

and

$$\delta_2^*(w,i) = -\frac{\delta_1^*(w,i)}{\beta}. \tag{4}$$

By Assumption 2, such a δ^* exists. Observe that conditions (1) and (4) guarantee that transfers sum to zero in periods one and two, respectively. Thus, aggregate feasibility is satisfied.

Next, one has to show that non-negative consumption is feasible for each agent under the proposed allocation. Observe that the NPV of transfers of any agent is equal to zero in this allocation. In period one, a poor agent with an idea faces the budget $c_1 + k_1 + s_1 \leq \bar{k}$ and chooses $k_1^*(p,1) = \bar{k}$. In period two in the low-return state, her consumption is $c_2^*(p,1,\theta_l) = f(\bar{k},\theta_l) - \frac{\bar{k}-p}{\beta} \geq 0$ by assumption. This clearly implies that $c_2(p,1,\theta_h) \geq 0$, too. Condition (2) guarantees that $(r,1)$ agent can choose investment equal to $\bar{k}$ and still consume a non-negative amount in period one. The consumption levels of agents without ideas are non-negative in both periods by (3). Obviously, given that they can sustain non-negative consumption without using the wasteful technology, no agent in the economy sets $s_1(w,i) > 0$ in the constrained efficient allocation.

The only thing left is to check that given δ^* agents will tell the truth about their types, but this is straightforward given that the NPV of transfers of any type is equal to zero.

B Proof of Lemma 2

Observe that $\Delta^*(p,1) \leq \bar{\Delta}$ implies that $\delta_1^*(p,1) \leq \bar{k} - p$. If not, then by Assumption 4, we have $c_2^*(p,1,\theta_l) < 0$, a contradiction. Now I show that when $\delta_1^*(p,1) \leq \bar{k} - p$, then incentive compatibility implies $c_1^*(p,1) = s_1^*(p,1) = 0$, and hence, $\delta_1^*(p,1) = k_1^*(p,1) - p$. First, suppose for contradiction that $c_1^*(p,1) > 0$. That implies $k_1^*(p,1) < \bar{k}$. Then, $(p,1)$ agent can decrease her consumption by a small amount in the first period and increase her investment by the same amount. This new allocation increases the welfare of $(p,1)$ strictly since her investment level was strictly below $\bar{k}$ when $c_1^*(p,1) > 0$, which implies the original allocation cannot be incentive compatible, a contradiction. By the same logic, $s_1^*(p,1) = 0$ as well.

Now, suppose that $\delta_2^*(p,1) > -f(k_1^*(p,1),\theta_l)$. First, observe that, this implies $k_1^*(p,1) < \bar{k}$, since if $k_1^*(p,1) = \bar{k}$, then we would have $\Delta^*(p,1) = \bar{k} - p + \beta\delta_2^*(p,1) > \bar{k} - p - \beta f(\bar{k},\theta_l) = \bar{\Delta}$, which contradicts with $\Delta^*(p,1) \leq \bar{\Delta}$. Second, define a new allocation with transfers $\tilde{\delta}_2(p,1) = \delta_2^*(p,1) - \epsilon$ and $\tilde{\delta}_1(p,1) = \delta_1^*(p,1) + \beta\epsilon$, and $\tilde{\delta}_2(r,1) = \delta_2^*(r,1) + \frac{\zeta_p}{\zeta_r}\epsilon$ and $\tilde{\delta}_1(r,1) = \delta_1^*(r,1) - \frac{\zeta_p}{\zeta_r}\beta\epsilon$, and the rest of the transfers remain unchanged. The resulting allocation is incentive compatible and feasible since the NPV of transfers is unchanged for all agents. For agents $(w,i) \neq (p,1)$, welfare is unchanged. To see the change in $(p,1)$'s welfare first observe that being investment constrained in the first period, $(p,1)$ will use all of the extra $\beta\epsilon$ units of period one transfers for investment. This, then, changes her welfare by

$$\underbrace{\beta\epsilon[g'(k_1^*(p,1))E\{\theta\} + (1-\kappa)]}_{\text{Gain in pd. 2}} - \underbrace{\epsilon}_{\text{Loss in pd. 2}} > 0,$$

since $k_1^*(p,1) < \bar{k}$. This means the new allocation improves over the constrained efficient one, implying a contradiction.

C Proof of Proposition 3

Suppose for contradiction that $\Delta^*(p,1) < 0$. Let $k_1^*(p,1) = k$. Since $\Delta^*(p,1) < 0 < \bar{\Delta}$, obviously, $k < \bar{k}$. By Lemma 2,

$$\delta_1^*(p,1) = k - p \text{ and } \delta_2^*(p,1) = -f(k,\theta_l)$$

and $\Delta^*(p,1) < 0$ implies

$$k - p \ - \beta f(k,\theta_l) < 0. \tag{5}$$

Now, consider a new allocation in which

$$\delta_1(p,1) = \beta f(k,\theta_l) \text{ and } \delta_2(p,1) = -f(k,\theta_l).$$

and the rest of the transfers are constructed such that NPV of transfers going to all agents $(w,i) \neq (p,1)$ are zero, total transfers sum up to zero every period, and all agents consumption are weakly positive. Observe that under this allocation, NPV of transfers going to $(p,1)$ is zero. Therefore, $k_1(p,1) < \bar{k}$, since by Assumption 4, in order to make (p,1) invest $\bar{k}$, we need to give her $\bar{\Delta} > 0$. Therefore, $(p,1)$ agent will use all of her period one resources to invest in her business:

$$k_1(p,1) = p + \beta f(k,\theta_l).$$

Observe that

$$k_1(p,1) = p + \beta f(k,\theta_l) > k,$$

where the inequality follows from (5). This implies that agent (p,1) has strictly higher welfare. Some agent will have lower welfare, but since (p,1)'s investment now is closer to the efficient level, the total change in welfare is strictly positive. Importantly, this new allocation is also incentive compatible since NPV of transfers going to all are zero. But this means the new allocation improves over the constrained efficient allocation, a contradiction.

D Proof of Proposition 4

Lemma 2 shows that the higher is the NPV of transfers that the poor agents with ideas receive the higher is their level of investment, and hence, the closer the society is to full information efficiency. As a result, the planner's problem is equivalent to maximizing the NPV of transfers going to poor agents with ideas subject to non-negativity of consumption and incentive feasibility. The proof proceeds in 3 steps. First, I show that storage technology is not used by anyone in the constrained efficient allocation. Second, I show that incentive compatibility implies that the NPV of transfers going to agents with same i type should be independent of wealth. This implies that all agents with ideas will be receiving the same transfers and all agents without ideas will be contributing the same amount to the financing of those transfers. In the last step, we analyze the final incentive compatibility condition that might limit planner's ability carry out required transfers: the incentive constraints regarding the deviation in which the agents without ideas pretend to have ideas. Analyzing that condition allows us to provide a full characterization of the constrained efficient allocation.

Step 1. $s_1^*(w,i) = 0$ for all $(w,i) \in W \times I$.

First, suppose for a contradiction that $s_1^*(w^o, i^o) > 0$, for some $(w^o, i^o) \in W \times I$. Now define a new allocation $(\tilde{c}, \tilde{k}, \tilde{s}, \tilde{\delta})$ that is identical to the constrained efficient allocation, except for

$$\begin{aligned}
\tilde{s}_1(w^o, i^o) &= s_1^*(w^o, i^o) - \epsilon, \\
\tilde{k}_1(p,1) &= k_1^*(p,1) + \epsilon \frac{\zeta_{w^o}\eta_{i^o}}{\zeta_p \eta_1}, \\
\tilde{c}_2(w,i,\theta) &= c_2^*(w,i,\theta) + \epsilon \frac{\zeta_{w^o}\eta_{i^o}}{\zeta_p \eta_1}\Big[\frac{1}{\beta} - A\Big].
\end{aligned}$$

The new allocation satisfies aggregate feasibility in period one by construction. Remember that $k_1^*(p,1) \leq \bar{k}$ in the constrained efficient allocation. Furthermore, we know from the full information analysis that the average marginal returns to investment at the full information level, $\bar{k}$, is equal to $\frac{1}{\beta}$. Under diminishing marginal returns to capital, these imply that the marginal returns to investing additional $\epsilon\frac{\zeta_{w^o}\eta_{i^o}}{\zeta_p \eta_1}$ units in poor agents with ideas is at least $\frac{1}{\beta}\epsilon\frac{\zeta_{w^o}\eta_{i^o}}{\zeta_p \eta_1}$ units, for ϵ small. The loss from saving less is $A\epsilon\frac{\zeta_{w^o}\eta_{i^o}}{\zeta_p \eta_1}$. Thus, the new consumption allocation satisfies period two aggregate feasibility condition and the planner might even be left with some extra resources. In the new allocation, period one consumption levels are identical to their levels in the original allocation and period two consumption levels increase by the same amount for all agents. Thus, if the original allocation is incentive compatible, the new allocation has to be incentive compatible as well. We have just shown that the new allocation is in the constraint set of the planner and provides all agents with higher welfare which means the original allocation cannot be constrained efficient, a contradiction.

Step 2. Now, I show that $\Delta^*(p,1) = \Delta^*(r,1)$. If $\Delta(p,1)^* > \Delta(r,1)^*$, then $(r,1)$ lies to be $(p,1)$ and gets the transfers with higher NPV; therefore, this cannot be true. On the other hand, if $\Delta(p,1)^* < \Delta(r,1)^*$, then one can propose a new allocation with a transfer system $\tilde{\delta}$ that is the same as δ^*, except for $\tilde{\delta}_1(p,1) = \delta_1^*(p,1) + \epsilon$ and $\tilde{\delta}_1(r,1) = \delta_1^*(r,1) - \frac{\zeta_p}{\zeta_r}\epsilon$. Clearly, this transfer mechanism is a part of a feasible allocation. This new allocation is also incentive compatible: $(r,1)$ does not lie to be $(p,1)$ since $\epsilon > 0$ is small. Agents without ideas do not lie to be $(p,1)$ since with original transfers they were not lying to be $(r,1)$ and the NPV of transfers of $(p,1)$ under the new mechanism is strictly lower than that of $(r,1)$ under the original transfer mechanism. But in the allocation that is attained by the new transfer mechanism each $(p,1)$ agent's utility increases strictly more than ϵ since he was investment-constrained under δ^*. The utility of each $(r,1)$ agent decreases by $\frac{\zeta_p}{\zeta_r}\epsilon$ and the utility of agents without ideas do not change. Summing these utility changes over all agents, we get that the new mechanism brings strictly higher total welfare. As a result, the original transfer mechanism cannot be constrained efficient, which is a contradiction. Now that we established $\Delta^*(p,1) = \Delta^*(r,1)$, by Assumption 5.b and without loss of generality, we set $\delta_1^*(r,1) = k_1^*(p,1) - p$ and $\delta_2^*(r,1) = -f(k_1^*(p,1), \theta_l)$. One can similarly show that in the constrained efficient allocation $\Delta^*(p,0) = \Delta^*(r,0)$.

Step 3. By aggregate feasibility, agents without ideas have to finance the transfers going to agents with ideas:

$$\Delta^*(w,0) = -\frac{\eta_1}{\eta_0}\Delta^*(w,1). \tag{6}$$

The only incentive compatibility condition that is left to check is the one regarding deviations in which an agent without an idea lies to have an idea. The planner increases $\Delta(w,1)$ until the incentive constraint of the agents without ideas bind. If this incentive constraint does not bind and $\Delta(w,1) = \bar{\Delta}$ is reached, then the planner stops increasing $\Delta(w,1)$ since full information efficient allocation has been reached. The incentive compatibility condition that the allocation that is a candidate to constrained efficiency has to satisfy for all w, w' is

$$w + \Delta(w,0) \geq \begin{cases} w + \Delta(w',1) + \frac{-\delta_2(w',1)}{A}[-1+\beta A], & \text{if } A(\delta_1^*(w',1) + w) \geq -\delta_2^*(w',1); \\ -\infty, & \text{if else.} \end{cases} \tag{7}$$

Here, the left-hand side of the equation is the utility of truth-telling, whereas the right-hand side is the utility of lying to be $(w',1)$. First, consider the right-hand-side of (7). For agents without ideas, the benefit of lying to have an idea is receiving the NPV of transfers agents with ideas receive, which is $\Delta(w',1)$. There is a cost of lying, too, however. When $(w,0)$ types lie, they have to set $s_1' \geq \frac{-\delta_2(w',1)}{A}$ in order to have non-negative consumption in period two. If $A(\delta_1(w',1) + w) \geq -\delta_2(w',1)$, the cost of lying is $\frac{-\delta_2(w',1)}{A}[-1+\beta A]$. If else, that means even if $(w,0)$ agents save all of their net of transfers wealth in period one, they still cannot generate $-\delta_2(w',1)$, which means they have to consume a negative amount either in period one or period two. In that case, the cost of lying is negative infinity.

Second, consider the left-hand side of (7). When agents of type $(w,0)$ tell the truth, they pay resources through period one transfers and receive resources through period two transfers. Thanks to Assumption 5.b, the planner can generate enough transfers from agents without ideas without making any of them consume a negative amount. Since they are receiving transfers in period two, agents without ideas do not have to use the wasteful saving technology when they tell the truth. Hence, the only cost of telling the truth is paying $\Delta(w,0)$ units in terms of NPV of transfers.

Remember from Lemma 2 that $\delta_1^*(w,1) = k_1^*(p,1) - p$ and $\delta_2^*(w,1) = -f(k_1^*(p,1),\theta_l)$. Plugging these in $A(\delta_1(w',1)+w) < -\delta_2(w',1)$, it follows that $k_1^*(p,1)$ units of investment for poor agents with ideas is attained in the constrained efficient allocation if, for any $w \in W$, we have:

$$A < \frac{f(k_1^*(p,1),\theta_l)}{k_1^*(p,1) - p + w}.$$

Alternatively, even if $A(\delta_1(w',1)+w) \geq -\delta_2(w',1)$, $k_1^*(p,1)$ units of investment for poor agents ideas is attained in the constrained efficient allocation if, for any $w \in W$, we have:

$$A \leq \frac{f(k_1^*(p,1),\theta_l)}{k_1^*(p,1) - p - \Delta^*(w,0)},$$

which follows from plugging Lemma 2 levels of transfers into the first line of (7).

In sum, $k_1^*(p,1)$ units of investment for poor agents with ideas is attained in the constrained efficient allocation if and only if it satisfies, for any $w \in W$:

$$A < \frac{f(k_1^*(p,1),\theta_l)}{k_1^*(p,1) - p + w}, \text{ or } A \leq \frac{f(k_1^*(p,1),\theta_l)}{k_1^*(p,1) - p - \Delta^*(w,0)}.$$

It follows from Assumption 5.a and aggregate feasibility that $\frac{f(k_1^*(p,1),\theta_l)}{k_1^*(p,1)-p+w} < \frac{f(k_1^*(p,1),\theta_l)}{k_1^*(p,1)-p-\Delta^*(w,0)}$. Hence, $k_1^*(p,1)$ is attained at the constrained efficient allocation if and only if $A \leq \frac{f(k_1^*(p,1),\theta_l)}{k_1^*(p,1)-p-\Delta^*(w,0)}$. Using equation (6), it follows that $k_1^*(p,1)$ is attained in constrained efficient allocation if and only if

$$A \leq \frac{\eta_0 f(k_1^*(p,1),\theta_l)}{k_1^*(p,1) - p - \eta_1\beta f(k_1^*(p,1),\theta_l)}. \tag{8}$$

Hence, for $A \leq \bar{A}$, as it is defined in Proposition 4, $k_1^*(p,1) = \bar{k}$ is incentive compatible. The feasibility of transfers for agents without ideas follow from Assumption 5.a and 5.b.

To see that for $A > \bar{A}$, $k_1^*(p,1)$ is given by $A = \frac{\eta_0 f(k_1^*(p,1),\theta_l)}{k_1^*(p,1)-p-\eta_1\beta f(k_1^*(p,1),\theta_l)}$, first observe that the expression on the right-hand side of equation (8) is strictly decreasing in k_1. Now suppose for a contradiction that $A < \frac{\eta_0 f(k_1^*(p,1),\theta_l)}{k_1^*(p,1)-p-\beta\eta_1 f(k_1^*(p,1),\theta_l)}$. Then, define a new transfer system $\tilde{\delta}$ which is identical to the constrained efficient one, δ^*, except for $\tilde{\delta}_1(w,1) = \delta_1^*(w,1) + \epsilon$ and $\tilde{\delta}_1(w,0) = \delta_1^*(w,0) - \frac{\eta_1}{\eta_0}\epsilon$, $\epsilon > 0$. By Lemma 2, this means the investment level for the poor agents with ideas is $\tilde{k}_1 = k_1^*(p,1) + \epsilon$. This decreases the right-hand side of equation (8). However, for ϵ small, equation (8) still holds under the new allocation. Thus, this new allocation is incentive compatible. It is clearly feasible. Finally, it strictly increases total welfare since it increases $(p,1)$ agents' investment increases. Then, δ^* cannot be constrained efficient, a contradiction.

Relaxing Assumption 5.

Assumption 5.a. $\eta_1 \bar{\Delta} \leq \eta_0 p$.

If Assumption 5.a does not hold, then full information efficient allocation cannot be attained even if $A \leq \bar{A}$. To see this, observe that the total amount of NPV of transfers that the planner needs to send to agents with ideas is $\eta_1 \bar{\Delta}$. On the other hand, by individual feasibility, the maximum amount of resources that can be taken from a poor agent without an idea without making him consume a negative amount is p. By incentive compatibility, that is also the maximum amount that can be taken from a rich agent without an idea. This means that the total amount that can be transferred from agents without ideas is $\eta_0 p$. Thus, if Assumption 5.a is violated, it is not possible achieve full information efficiency even if $A \leq \bar{A}$. In this case, the NPV of transfers to agents with ideas would be given by $\frac{\eta_0 p}{\eta_1}$. And if $A > \bar{A}$, then constrained efficient level of transfer going to agents with ideas would be determined by the incentive compatibility condition of agents without ideas holding with equality, as before. Therefore, the message of Proposition 4 would essentially be unchanged, only one more case would be added. Since whether full information efficiency is reached or not is only a side issue to the current paper, I chose to make Assumption 5.a and keep this extra case out of the paper.

Assumption 5.b. $\eta_1(\bar{k} - p) \leq \eta_0 \sum_w \zeta_w w$.

In step 2 of the proof of Proposition 4, without loss of generality, we set $\delta_t^*(r,1) = \delta_t^*(p,1)$ for $t = 1, 2$. With this restriction on the allocation, the planner needs to raise $\eta_1(\bar{k}-p)$ units of resources to transfer to agents with ideas in period one. Assumption 5.b. ensures that if the planner collects all the wealth of agents without ideas in period one, $\eta_0 \sum_w \zeta_w w$, that would be enough to finance the transfers to agents with ideas, $\eta_1(\bar{k} - p)$. Without this assumption, under the restriction of $\delta_t^*(r,1) = \delta_t^*(p,1)$ for $t = 1, 2$ on allocations space, we cannot reach full information efficiency even if $A \leq \bar{A}$. Again since whether full information efficiency is reached or not is only a side issue to the current paper, I chose to make Assumption 5.a and keep this extra case out of the paper.

E Proof of Corollary 5

For all $w \in W$,

$$\Delta^*(w,1) = k_1^*(p,1) - p - \beta f(k_1^*(p,1), \theta_l). \tag{9}$$

If $A < \bar{A}$, then $k_1^*(p,1) = \bar{k}$, and $\Delta^*(w,1) = \bar{\Delta} > 0$, by Assumption 4. If $A \geq \bar{A}$, then the implicit equation giving $k_1^*(p,1)$ implies that

$$\begin{aligned} k_1^*(p,1) - p - \eta_1 \beta f(k_1^*(p,1), \theta_l) &= \frac{\eta_0 f(k_1^*(p,1), \theta_l)}{A} \\ &> \eta_0 \beta f(k_1^*(p,1), \theta_l), \end{aligned}$$

where the inequality follows from Assumption 1, that is $A\beta < 1$. Then, collecting all the terms to the left gives

$$\Delta^*(w,1) = k_1^*(p,1) - p - \beta f(k_1^*(p,1), \theta_l) > 0.$$

F Proof of Proposition 7

Since the allocation described attains productive efficiency, we only need to check that it is incentive compatible and satisfies aggregate and individual feasibility conditions with no agent consuming a negative amount. Incentive compatibility directly follows from the fact that the NPV of transfers is zero for each agent. Aggregate and individual feasibility is by construction. That each agent consumes a non-negative amount in any period is obvious except for $(p, 1)$ agent in θ_h state. So, we need to show that $\delta_2(p, 1, \theta_h) \geq -f(\bar{k}, \theta_h)$.

For a contradiction, suppose that $\delta_2(p, 1, \theta_h) < -f(\bar{k}, \theta_h)$. By construction, $\delta_2(p, 1, \theta_h)$ satisfies

$$\bar{k} - p + \beta[-\mu_l f(\bar{k}, \theta_l) + \mu_h \delta_2(p, 1, \theta_h)] = 0.$$

Therefore, we get:

$$\bar{k} - p > \beta[E\{\theta\}g(\bar{k}) + (1 - \kappa)\bar{k}]. \tag{10}$$

Remember that the first-order condition that gives $\bar{k}$ is:

$$1 = \beta[g'(\bar{k})E\{\theta\} + (1 - \kappa)].$$

Multiplying both sides by $\bar{k}$ and then subtracting p from both sides gives

$$\bar{k} - p = \beta[g'(\bar{k})\bar{k}E\{\theta\} + (1 - \kappa)\bar{k}] - p.$$

Concavity of the function $g(\cdot)$ implies $g'(k)k < g(k)$. This, with $p > 0$, imply that

$$\bar{k} - p < \beta[g'(\bar{k})\bar{k}E\{\theta\} + (1 - \kappa)\bar{k}],$$

which contradicts with (10).

G Proof of Proposition 8

Suppose for contradiction that the constrained efficient allocation can be achieved. Then, $b_1(p, 1) \leq -(\bar{k} - p)$. Due to linearity of preferences, $R = 1/\beta$. Thus, $c_2(p, 1, \theta_l) \leq \theta_l \bar{k}^\alpha - \frac{\bar{k}-p}{\beta} < 0$, by Assumption 3. But this cannot be an optimal choice for the agent since the agent could do better just by setting $b_1(p, 1) = 0$. Thus, we have a contradiction.

H Proof of Proposition 9

Now I construct a market equilibrium where $R = 1/\beta$ and agents with ideas invest at their corresponding constrained efficient investment level.

Under the specified taxes, an agent faces the following problem:

Agent's problem with taxes.

$$\max_{c,k,b,s} u(c_1) + \beta \sum_{\theta} \mu_\theta u(c_{2\theta})$$

s.t.

$$c_1 + k_1 + b_1 + s_1 \leq \begin{cases} w + \Delta^*(w,1), & \text{if } b_1 \leq -\beta f(k_1^*(p,1), \theta_l) \\ w - \frac{\eta_1}{\eta_0}\Delta^*(w,1), & \text{if else,} \end{cases}$$

$$c_{2\theta} \leq f(k_1, \theta) + Rb_1 + As_1,$$
$$s_1, k_1 \geq 0.$$

First, consider an agent who has an idea in period one. If a poor agent with an idea chooses $b_1 \leq -\beta f(k_1^*(p,1), \theta_l)$, she chooses $k_1 = k_1^*(p,1)$ and $b_1 = p - k_1^*(p,1) + \Delta^*(w,1) = -\beta f(k_1^*(p,1), \theta_l)$. Suppose for contradiction that this is not true. Then, there is (k_1', b_1', s_1'), where $(k_1', s_1') \neq (k_1^*(p,1), 0)$, which gives strictly greater utility to the agent. $s_1' = 0$ follows immediately from the fact that the return to bonds is strictly greater than the risk-free return; hence, it has to be that $k_1' \neq k_1^*(p,1)$. Then, define a new allocation with transfers $\delta_1'(w,1) = k_1' - p$, $\delta_2'(w,1) = -\frac{k_1' - p - \Delta^*(w,1)}{\beta}$, and $(\delta_1'(w,0), \delta_2'(w,0))_{w \in W}$ such that $\Delta'(w,0) = -\eta_1/\eta_0 \Delta'(w,1)$ and individual consumption levels are non-negative. $(p,1)$ chooses (k_1', b_1') in the market, implying that she chooses k_1' in the planner's problem when she faces δ'. The only thing left to check is incentive compatibility. That holds because of the way in which the new allocation is constructed, $\Delta'^*(w,1)$. Therefore, this new allocation is incentive compatible and feasible, keeps the welfare of $(w,i) \neq (p,1)$ unchanged compared to the constrained efficient allocation, and provides strictly greater welfare than the constrained efficient level for poor agents with ideas. This means that the new allocation is an improvement over the constrained efficient allocation, a contradiction.

Similarly, one can show that when $b_1(r,1) \leq -\beta f(k_1^*(p,1), \theta_l)$, $(r,1)$ agent chooses to invest at the constrained efficient level, $\bar{k}$.

Now we need to show that agents with ideas choose $b_1 \leq -\beta f(k_1^*(p,1), \theta_l)$. The utility of $(w,1)$ type when she chooses $b_1 \leq -\beta f(k_1^*(p,1), \theta_l)$, is:

$$\begin{aligned} & w + \Delta^*(w,1) - k_1^*(w,1) - b_1(w,1) + \beta \sum_\theta \mu_\theta [f(k_1^*(w,1), \theta) + b_1(w,1)/\beta] \\ & = w + \Delta^*(w,1) - k_1^*(w,1) + \beta \sum_\theta \mu_\theta f(k_1^*(w,1), \theta). \end{aligned} \tag{11}$$

On the other hand, if an agent with an idea chooses $b_1 > -\beta f(k_1^*(p,1), \theta_l)$, then, letting her optimal choices be $\tilde{k}_1(w,1), \tilde{b}_1(w,1)$, her utility would be:

$$\begin{aligned} & w - \frac{\eta_1}{\eta_0}\Delta^*(w,1) - \tilde{k}_1(w,1) - \tilde{b}_1(w,1) + \beta \sum_\theta \mu_\theta [f(\tilde{k}_1(w,1), \theta) + \tilde{b}_1(w,1)/\beta] \\ & = w - \frac{\eta_1}{\eta_0}\Delta^*(w,1) - \tilde{k}_1(w,1) + \beta \sum_\theta \mu_\theta f(\tilde{k}_1(w,1), \theta). \end{aligned} \tag{12}$$

The difference between the maximized values of $(w,1)$ agents' problem in the market under $b_1 \leq -\beta f(k_1^*(p,1), \theta_l)$ and under $b_1 > -\beta f(k_1^*(p,1), \theta_l)$ then is given by subtracting (12) from (11) :

$$\frac{\Delta^*(w,1)}{\eta_0} + [-k_1^*(w,1) + \beta \sum_\theta \mu_\theta f(k_1^*(w,1), \theta)] - [-\tilde{k}_1(w,1) + \beta \sum_\theta \mu_\theta f(\tilde{k}_1(w,1), \theta)]. \tag{13}$$

Given that $\bar{k}$ maximizes the function $-k + \beta \sum_\theta \mu_\theta f(k, \theta)$, it is obvious that this difference is strictly positive for $(r, 1)$. For $(p, 1)$, under $b_1 > -\beta f(k_1^*(p, 1), \theta_l)$, $\tilde{k}_1(w, 1) < k_1^*(p, 1) \leq \bar{k}$. This, combined with the fact that the function $-k + \beta \sum_\theta \mu_\theta f(k, \theta)$ is strictly increasing in k, for $k \leq \bar{k}$, implies that the expression in (13) is also strictly positive. Hence, we showed that agents with ideas act according to the constrained efficient allocation in the market.

Now consider agents who do not have an idea in period one. If they choose $b_1 \leq -\beta f(k_1^*(p, 1), \theta_l)$, then $c_2(w, 0) \leq -f(k_1^*(p, 1), \theta_l) + As_1(w, 0)$. To keep consumption non-negative, $s_1(w, 0) \geq \frac{f(k_1^*(p,1), \theta_l)}{A}$. Since $A < \beta^{-1}$, these agents will invest as little as possible in risk-free technology. This implies they choose $b_1 = -\beta f(k_1^*(p, 1), \theta_l)$ and $s_1(w, 0) = \frac{f(k_1^*(p,1), \theta_l)}{A}$. The utility then is $w + \Delta^*(w, 1) + \beta f(k_1^*(p, 1), \theta_l) - \frac{f(k_1^*(p,1), \theta_l)}{A}$.

When an agent with no ideas chooses $b_1 > -\beta f(k_1^*(p, 1), \theta_l)$, she sets $s_1 = 0$ and chooses the constrained efficient allocation. The utility she gets is $w - \frac{\eta_1}{\eta_0}\Delta^*(w, 1)$.

We need to show that, for agents without ideas, utility under $b_1 > -\beta f(k_1^*(p, 1), \theta_l)$ is greater than utility under $b_1 \leq -\beta f(k_1^*(p, 1), \theta_l)$. The difference is equal to

$$
\begin{aligned}
& w - \frac{\eta_1}{\eta_0}\Delta^*(w, 1) - [w + \Delta^*(w, 1) + \beta f(k_1^*(p, 1), \theta_l) - \frac{f(k_1^*(p, 1), \theta_l)}{A}] \\
&= \frac{-\Delta^*(w, 1)}{\eta_0} - f(k_1^*(p, 1), \theta_l)(\beta - 1/A) \\
&= 0,
\end{aligned}
$$

where the last inequality follows from $A = \frac{\eta_0 f(k_1^*(p,1), \theta_l)}{k_1^*(p,1) - p - \eta_1 \beta f(k_1^*(p,1), \theta_l)}$.

Market clearing and government budget balance conditions are immediate from the fact that the constrained efficient allocation satisfies aggregate feasibility and has non-negative consumption for all agents.